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Effects of a Preschool Mathematics Curriculum: Summative Research on the *Building Blocks* Project

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This study evaluated the efficacy of a preschool mathematics program based on a comprehensive model of developing research-based software and print curricula. *Building Blocks*, funded by the National Science Foundation, is a curriculum development project focused on creating research-based, technology-enhanced mathematics materials for pre-K through grade 2. In this article, we describe the underlying principles, development, and initial summative evaluation of the first set of resulting materials as they were used in classrooms with children at risk for later school failure. Experimental and comparison classrooms included two principal types of public preschool programs serving low-income families: state funded and Head Start prekindergarten programs. The experimental treatment group score increased significantly more than the comparison group score; achievement gains of the experimental group approached the sought-after 2-sigma effect of individual tutoring. This study contributes to research showing that focused early mathematical interventions help young children develop a foundation of informal mathematics knowledge, especially for children at risk for later school failure.

Key words: Computers, Curriculum, Early childhood, Equity/diversity, Instructional intervention, Instructional technology, Preschool/primary, Program/project assessment

Curricula are rarely developed or evaluated scientifically (Clements, 2007). Less than 2% of research studies in mathematics education have concerned the effects of textbooks (Senk & Thompson, 2003). This study is one of several coordinated efforts to assess the efficacy of a scientifically based curriculum; specifically, whether a preschool mathematics curriculum was developing the mathematical knowledge of disadvantaged 4-year-old children (Clements, 2002; Clements & Battista, 2000).

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Building Blocks is a NSF-funded pre-K to grade 2 mathematics curriculum development project designed to comprehensively address recent standards for early mathematics education for all children (e.g., Clements, Sarama, & DiBiase, 2004; NCTM, 2000). Previous articles describe the design principles behind a set of research-based software microworlds included in the *Building Blocks* program and the research-based design model that guided its development (Clements, 2002; Clements & Sarama, 2004a; Sarama & Clements, 2002). This article presents initial summative research on the first set of resulting materials: a research-based, technology-enhanced preschool mathematics curriculum.

There have been few rigorous tests of the effects of preschool curricula. Although some evidence indicates that curriculum can strengthen the development of young students' knowledge of number or geometry (Clements, 1984; Griffin & Case, 1997; Razel & Eylon, 1991), no studies of which we are aware have studied the effects of a complete preschool mathematics curriculum, especially on low-income children, children who are at serious risk for later failure in mathematics (Bowman, Donovan, & Burns, 2001; Campbell & Silver, 1999; Secada, 1992). These children possess less mathematical knowledge than higher-income children even before first grade (Denton & West, 2002; Ginsburg & Russell, 1981; Griffin, Case, & Capodilupo, 1995; Jordan, Huttenlocher, & Levine, 1992; Klein & Starkey, 2004). They also receive less support for mathematics learning in the home and school environments, including preschool, than their higher-income peers (Blevins-Knabe & Musun-Miller, 1996; Bryant, Burchinal, Lau, & Sparling, 1994; Farran, Silveri, & Culp, 1991; Holloway, Rambaud, Fuller, & Eggers-Pierola, 1995; Saxe, Guberman, & Gearhart, 1987; Starkey et al., 1999).

DESIGN OF THE *BUILDING BLOCKS* MATERIALS

Many curriculum and software publishers claim a research basis for their materials, but the bases of these claims are often dubitable (Clements, 2002). The *Building Blocks* project is based on the assumption that research-based curriculum development efforts can contribute to (a) more effective curriculum materials, (b) better understanding of students' mathematical thinking, and (c) research-based change in mathematics curricula (Clements, Battista, Sarama, & Swaminathan, 1997; Schoenfeld, 1999). Indeed, along with our colleagues, we believe that education will not improve substantially without a systemwide commitment to research-based curriculum and software development (Battista & Clements, 2000; Clements, 2007; Clements & Battista, 2000).

Our theoretical foundation for research-based curriculum development and evaluation, the Curriculum Research Framework (CRF), comprises three categories spanned by 10 phases, summarized in Table 1 (Clements, 2002; see especially Clements, 2007). Given the comprehensive CRF, claims that a curriculum is based on research should be questioned to reveal the exact nature between the curriculum and the research used or generated. Unfortunately, there is little documentation of the phases used for most curricula. Often, there is only a hint of *A Priori Foundations*

Table 1
Categories and Phases of Curriculum Research (adapted from Clements, 2007)

Categories	Phases	Description of knowledge gained
A Priori Foundations. Extant research is reviewed, and implications for the nascent curriculum development drawn.	1. Subject Matter A Priori Foundation	Description of specific subject matter content, including the role it would play in students' development
	2. General A Priori Foundation	Relevant information from psychology, education, and systemic change
	3. Pedagogical A Priori Foundation	Instruction, including the effectiveness of certain types of activities
Learning Model. Activities are structured based on empirical models.	4. Structure According to Specific Learning Models	Children's mathematical thinking and learning and correlated activities constituting specific learning trajectories
Evaluation. Empirical evidence is collected to evaluate the appeal, usability, and effectiveness of an instantiation of the curriculum.	5. Market Research	Marketability
	6. Formative Research: Small Group 7. Formative Research: Single Classroom 8. Formative Research: Multiple Classrooms	Meanings students and teachers give to the curriculum objects and activities in progressively expanding social contexts, and the usability and effectiveness of specific components and characteristics of the curriculum. The curriculum is altered based on empirical results, including support for teachers.
	9. Summative Research: Small Scale 10. Summative Research: Large Scale	Experimental evaluation, include fidelity and sustainability of the curriculum when implemented on a small, then large, scale, and the critical contextual and implementation variables that influence its effectiveness

phases, sometimes nonscientific market research, and minimal formative research with small groups. For example, "beta testing" of educational software is often merely polling of easily accessible peers, conducted late in the process, so that changes are minimal, given the time and resources dedicated to the project already and the limited budget and pressing deadlines that remain (Char, 1989; Clements & Battista, 2000). In contrast, we designed the *Building Blocks* approach to incorporate as many of the phases as possible.

Previous publications provide detailed descriptions of how we applied the CRF in our design process model (Clements, 2002; Sarama, 2004; Sarama & Clements, 2002); here we provide an overview of our application of the CRF. *A Priori Foundation* phases were used to determine the curriculum's goals and pedagogy. Based on theory and research on early childhood learning and teaching (Bowman et al., 2001; Clements & Sarama, in press), we determined that *Building Blocks'* basic approach would be *finding the mathematics in, and developing mathematics from, children's activity*. The materials are designed to help children extend and mathematize their everyday activities, from building blocks (the first meaning of the project's name) to art and stories to puzzles. Activities are designed based on children's experiences and interests, with an emphasis on supporting the development of *mathematical* activity. To do so, the materials integrate three types of media: computers, manipulatives (and everyday objects), and print. Pedagogical foundations were similarly established; for example, we reviewed research using computer software with young children (Clements, Nastasi, & Swaminathan, 1993; Clements & Swaminathan, 1995; Steffe & Wiegel, 1994). This research showed that computers can be used effectively by children as young as 3 or 4 years of age and that software can be made more motivating and educationally effective by, for example, using animation and children's voices and giving simple, clear feedback.

The phase of *Subject Matter A Priori Foundation* was used to determine subject matter content by considering what mathematics is culturally valued (e.g., NCTM, 2000) and empirical research on what constituted the core ideas and skill areas of mathematics for young children (Baroody, 2004; Clements & Battista, 1992; Fuson, 1997), with an emphasis on topics that were mathematical foundational, generative for, and interesting to young children (Clements, Sarama, et al., 2004). One of the reasons underlying the name we gave to our project was our desire that the materials emphasize the development of basic *mathematical building blocks* (the second meaning of the project's name)—ways of knowing the world mathematically—organized into two areas: spatial and geometric competencies and concepts, and numeric and quantitative concepts. Research shows that young children are endowed with intuitive and informal capabilities in both these areas (Baroody, 2004; Bransford, Brown, & Cocking, 1999; Clements, 1999; Clements, Sarama, et al., 2004). Three mathematical themes are woven through both these main areas: patterns, data, and sorting and sequencing. For example, challenging number activities do not just develop children's number sense; they can also develop children's competencies in such logical competencies as sorting and ordering (Clements, 1984).

Perhaps the most critical phase for *Building Blocks* was *Structure According to Specific Learning Model*. All components of the *Building Blocks* project are based on learning trajectories for each core topic. First, empirically based models of children's thinking and learning are synthesized to create a developmental progression of levels of thinking in the goal domain (Clements & Sarama, 2004b; Clements, Sarama, et al., 2004; Cobb & McClain, 2002; Gravemeijer, 1999; Simon, 1995). Second, sets of activities are designed to engender those mental processes or actions hypothesized to move children through a developmental progression. We present two examples, one in each of the main domains of number and geometry.

The example for number involves addition. Many preschool curricula and practitioners consider addition an inappropriate topic before elementary school (Clements & Sarama, in press; Heuvel-Panhuizen, 1990). However, research shows that children as young as toddlers can develop simple ideas of addition and subtraction (Aubrey, 1997; Clements, 1984; Fuson, 1992a; Groen & Resnick, 1977; Siegler, 1996). As long as the situation makes sense to them (Hughes, 1986), young children can directly model different types of problems using concrete objects, fingers, and other strategies (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Such early invention of strategies, usually involving concrete objects and based on subitizing and counting, plays a critical developmental role, as the sophisticated counting and composition strategies that develop later are all abbreviations or curtailments of these early solution strategies (Carpenter & Moser, 1984; Fuson, 1992a).

Most important for our purpose, reviews of research provide a consistent developmental sequence of the types of problems and solutions in which children can construct solutions (Carpenter & Moser, 1984; Clements & Sarama, in press; for the syntheses most directly related to our work, see Clements, Sarama, et al., 2004; Fuson, 1992a). Selected levels of the resulting addition learning trajectory are presented in Figure 1. The left column briefly describes each level and the research supporting it. The middle column provides a behavioral example illustrating that level of thinking. The learning trajectory continues beyond the last row in Figure 1 (details are in the *Building Blocks* curriculum and other sources, Clements & Sarama, 2007; Clements, Sarama, et al., 2004).

The next step of building the learning trajectory is to design materials and activities that embody actions on objects in a way that mirrors what research has identified as critical mental concepts and processes—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks are instantiated in on- and off-computer activity as actions (processes) on objects (concepts). For example, children might create, copy, and combine discrete objects, numbers, or shapes as representations of mathematical ideas. Offering students such objects and actions is consistent with the Vygotskian theory that mediation by tools and signs is critical in the development of human cognition (Steffe & Tzur, 1994). Further, designs based on objects and actions force the developer to focus on explicit actions or processes and what they will mean to the students. For example, on- and off-computer activity sets such as "Party Time" have the advantage of authenticity as well as serving as a way for children to mathematize these activities. In one of the "Party Time" activities involving setting the table, children use different mathematical actions such as establishing one-to-one correspondence, counting, and using numerals to represent and generate quantities to help get ready for a party. For these and other activities, the tasks themselves are often variations of those common in educational curriculum; what is unique in these cases is the more detailed consideration of actions on objects, the placement of the tasks in the research-guided learning trajectories, and the use of software.

For the addition trajectory, at the *Nonverbal Addition* level, children work on a software program in which they see three toppings on a pizza, then, after the top of the box closes, one more being placed on the pizza. Children put the same number of toppings on the other pizza (see the right column in the first row of Figure 1). The teacher conducts similar activities with children using colored paper pizzas and manipulatives for toppings. Similarly, the Dinosaur Shop scenario is used in several contexts. The teacher introduces a dinosaur shop in the sociodramatic play area and encourages children to count and add during their play. The *Small Number Addition* row in Figure 1 illustrates a task in which children must move dinosaurs in two boxes into a third and label the sum. Thus, the objects in

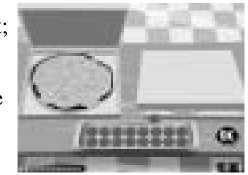
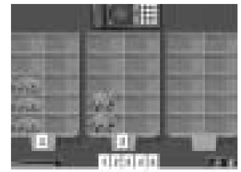
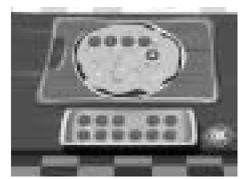
Level	Behavioral example	Instructional task
Nonverbal Addition. Children reproduce small (< 5) sums when shown the addition of subtraction of groups of objects (Baroody, 2004; Mix, Huttenlocher, & Levine, 2002).	After watching 2 objects, then 1 more placed under a cloth, children choose or make collections of 3 to show how many are hidden in all.	Two pepperoni slices were shown on the pizza on the right; at this point, the child sees 1 more moving out of the tray onto the pizza. Children put the same number of slices on the left pizza. 
Small Number Addition. Children solve simple "join, result unknown" word problems with sums to 5, usually by subitizing (instant identification of small collections) or using a "counting all" strategy (Baroody, 1987; Fuson, 1988).	"You have 2 balls and get 1 more. How many in all?" Child counts out 2, then counts out 1 more, then counts all 3.	The customer wants his order in one box; what should the label for that (rightmost) box be? 
Find Result. Children solve "join, result unknown" problems by direct modeling—"separating from" for subtraction or counting all for addition, with sums to 10 (Carpenter et al., 1993; Clements, Sarama, et al., 2004; Fuson, 1992a).	"You have 3 red balls and then get 3 blue balls. How many do you have in all?" Child counts out 3 red, then counts out 3 blue, then counts all 6.	Children play with toy dinosaurs on a background scene. For example, they might place 4 tyrannosaurus rex and then 5 apatosaurus on the paper and then count all 9 to see how many dinosaurs they have in all. 
Find Change. Children solve "change unknown" word problems by direct modeling. For example, they might "add on" to answer how many more blocks they would have to get if they had 4 blocks and needed 6 blocks in all (Clements, Sarama, et al., 2004).	"You have 5 balls and then get some more. Now you have 7 in all. How many did you get?" Child counts out 5, then counts those 5 again starting at one, then adds more, counting "6, 7," then counts the balls added to find the answer, 2.	Children are shown a pizza with 4 toppings but should have 7. They are asked to "make it 7." 
Counting On. Children continue developing their counting methods even further, often using objects to keep track. Such counting requires conceptually embedding the 3 inside the total, 5 (Baroody, 2004; Carpenter & Moser, 1984; Fuson, 1992b).	"How much is 4 and 3 more?" "Fourrrrr . . . 5 [putting up one finger], 6 [putting up a second finger], 7 [putting up a third finger]. Seven!"	Children use cutout "pizzas" and disks for toppings. The teacher asks them to put 5 toppings on their pizzas, and then asks how many they would have in all if they put on 3 more. They count on to answer, then actually put the toppings on to check.

Figure 1. Hypothetical learning trajectory for addition.

these and other tasks for the levels described in Figure 1 are single items, groups of items, and numerals. The actions include creating, duplicating, moving, combining, separating, counting, and labeling these objects and groups to solve tasks corresponding to the levels. The unique advantages of the software contexts include making these actions explicit, linking representations (computer manipulatives, spoken number words, and numerals), providing feedback, and guiding children along the research-based learning trajectories (e.g., moving a level forward or backward depending on a children's performance).

An example in geometry involves shape composition (other domains were shapes and their properties, transformations/congruence, and measurement, all determined through consensus building, see Clements, Sarama, et al., 2004). The composition of two-dimensional geometric figures was determined to be significant for students in two ways. First, it is a basic geometric competence, growing from preschoolers' building with shapes to sophisticated interpretation and analysis of geometric situations in high school mathematics and above. Second, the concepts and actions of creating and then iterating units and higher-order units in the context of constructing patterns, measuring, and computing are established bases for mathematical understanding and analysis (Clements et al., 1997; Reynolds & Wheatley, 1996; Steffe & Cobb, 1988). The domain is significant to research and theory in that there is a paucity of research on the trajectories students might follow in learning this content.

The developmental progression was born in observations of children's explorations (Sarama, Clements, & Vukelic, 1996) and refined through a series of clinical interviews and focused observations (leading to the learning trajectory summarized in Figure 2, adapted from Clements, Wilson, & Sarama, 2004). From a lack of competence in composing geometric shapes (*Pre-Composer)*, children gain abilities to combine shapes—initially through trial and error (e.g., *Picture Maker)* and gradually by attributes—into pictures, and finally synthesize combinations of shapes into new shapes (composite shapes). For example, consider the *Picture Maker* level in Figure 2. Unlike earlier levels, children concatenate shapes to form a component of a picture. In the top picture in that row, a child made arms and legs from several contiguous rhombi. However, children do not conceptualize their creations (parallelograms) as geometric shapes. The puzzle task pictured at the bottom of the middle column for that row illustrates a child incorrectly choosing a square because the child is using only one component of the shape, in this case, side length. The child eventually finds this does not work and completes the puzzle but only by trial and error.

A main instructional task requires children to solve outline puzzles with shapes off and on the computer, a motivating activity (Sales, 1994; Sarama et al., 1996). The software activity "Piece Puzzler" is illustrated in the third column in Figure 2 (on pages 144–145). The *objects* are shapes and composite shapes and the *actions* include creating, duplicating, positioning (with geometric motions), combining, and decomposing both individual shapes (units) and composite shapes (units of units). The characteristics of the tasks require actions on these objects corresponding to each level in the learning trajectory. Note that tasks in these tables are intended to support the developing of the *subsequent* level of thinking. That is, the instructional task in the *Pre-Composer* row is assigned to a child operating at the *Pre-Composer* level and is intended to facilitate the child's development of competencies at the *Piece Assembler* level.

Ample opportunity for student-led, student designed, open-ended projects are included in each set of activities. Problem posing on the part of students appears to be an effective way for students to express their creativity and integrate their learning (Brown & Walter, 1990; Kilpatrick, 1987; van Oers, 1994), although few empirical studies have been conducted, especially on young children. The computer can offer support for such projects (Clements, 2000). For "Piece Puzzler," students design their own puzzles with the shapes; when they click on a "Play" button, their design is transformed into a shape puzzle that either they or their friends can solve. In the addition scenarios, children can make up their own problems with pizzas and toppings, or dinosaurs and boxes.

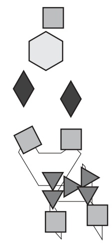
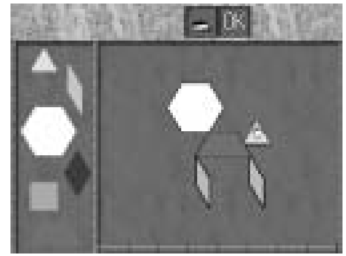
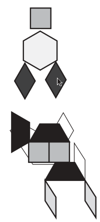
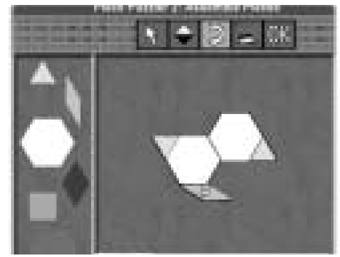
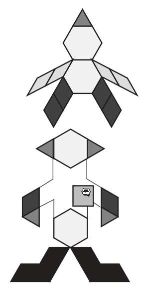
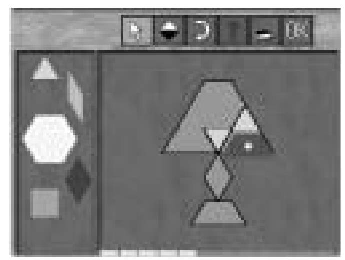
Our application of formative evaluation phases 5–8 is described in previous publications (Sarama, 2004; Sarama & Clements, 2002). In brief, we tested components of the curriculum and software using clinical interviews and observations of a small number of students to ascertain how children interpreted and understood the objects, actions, and screen design. Next, we tested whether children's actions on objects substantiated the actions of the researchers' model of children's mathematical activity, and we determined effective prompts to incorporate into each level of each activity. Although teachers were involved in all phases of the design, in phases 7–8 we focused on the process of curricular enactment (Ball & Cohen, 1996), using classroom-based teaching experiments and observing the entire class for information concerning the usability and effectiveness of the software and curriculum. Finally, a content analyses and critical review of the materials at each stage of development was conducted by the advisory board for the project.

In summary, we designed the *Building Blocks* materials in what we consider a well-defined, rigorous, and complete fashion, following the CRF. We built on previous curriculum efforts (Clements, 1984; Griffin & Case, 1997), but extended them into topics other than number and sequenced activities based on well-defined learning trajectories. The main purpose of this study was to evaluate whether materials created according to that model are effective in developing the mathematical knowledge of disadvantaged 4-year-old children and the size of that effect. A secondary purpose was to describe the degree to which the materials developed specific mathematics concepts and skills. To accomplish these two purposes, we used phase 9, *Summative Research: Small Scale*.

METHOD

Participants

Summative research was conducted at two sites, involving the two principal types of public preschool programs serving low-income families: state funded (site 1) and

Level	Examples (above, free-form pictures; below, puzzles)	Instructional task
Pre-Composer. Children manipulate shapes as individuals but are unable to combine them to compose a larger shape. In free form—“make a picture”—tasks, shapes often picture in middle do not touch (upper column). In puzzle tasks, shapes do not match simple outlines (lower picture in middle column). The instructional task (illustrated similar tasks on the computer in the last column; are presented with manipulatives and paper outlines or wooden form puzzles) uses outlines in which children can simply match shapes without turn or flip motions. (This and subsequent levels emerged from the same body of research, Clements, Wilson, et al., 2004; Mansfield & Scott, 1990; Sales & Hildebrandt, 2002; Sarama et al., 1996.)		
Piece Assembler. Children can place shapes contiguously to form pictures. In free-form tasks, each shape used represents a unique role or function in the picture (e.g., one shape for one leg). Children can fill only simple frames, in which shape is outlined, although their use of turns and flips is limited.		
Picture Maker. In free-form tasks, children can concatenate shapes to form pictures in which several shapes play a single role but use trial and error and do not anticipate creation of new geometric shapes. For puzzle tasks, children can match by a side length and use trial and error (a “pick and discard” strategy). Instructional tasks have “open” areas in which shape selection is ambiguous.		

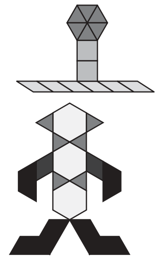
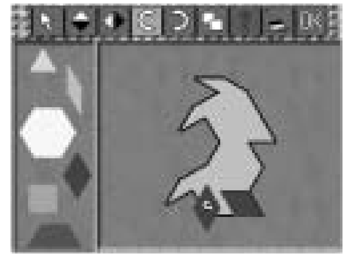
Level	Examples (above, free-form pictures; below, puzzles)	Instructional task
Shape Composer. Children combine shapes to make new shapes or fill puzzles, with growing intentionality and anticipation. Shapes are chosen using angles as well as side lengths. Eventually, the child considers several alternative shapes with angles equal to the existing arrangement. Instructional tasks (here, solving similar problems multiple ways) encourage higher levels in the hierarchy not described here, involve substitutions (higher levels are described in Clements, Wilson, et al., 2004).		

Figure 2. Hypothetical learning trajectory for shape composition (Clements, Wilson, et al., 2004).

Head Start (site 2) prekindergarten programs. State funded programs are urban programs in which most children receive free (63%) or reduced lunch (11%) and are 58% African American, 11% Hispanic, 28% White non-Hispanic, and 3% other. Head Start programs are urban programs in which virtually all children are qualified to receive free (97%) or reduced lunch (2%) and are 47% African American, 13% Hispanic, 30% White non-Hispanic, and 10% other. At each site, one classroom was assigned as experimental, one comparison. Both site 1 teachers had worked with us on the early development of the materials and were considered excellent teachers by their principal and peers. They agreed to have one selected to teach the *Building Blocks* materials and the other to continue using the school's curriculum until the following year. The experimental teacher at site 2 was inexperienced (2 years teaching), but she had an experienced aide; the comparison teacher had taught 8 years in the Head Start program. Neither of the site 2 teachers had worked with us previously. The two experimental teachers spent a half day with us viewing and discussing the materials.

All children in all four classes returned human subjects review forms. However, a total of 9 children moved out of the school during the year, 1 from the site 1 and 8 from site 2, leaving the following breakdown of children who participated in the pretest and completed at least one full section of the posttest: experimental—site 1, 6 boys, 11 girls, site 2, 7 boys, 6 girls; comparison—site 1, 9 boys, 7 girls, site 2, 13 boys, 9 girls. The average age of the 68 children at the time of pretesting was 49.9 months (SD = 6.2; range 34.8 to 57.8).

Design

We assessed the mathematics knowledge of all participating children at the beginning and again at the end of the school year. The experimental teachers implemented the *Building Blocks* preschool curriculum following the pretesting. This study is a component of the larger evaluation, which includes case studies of two students in each experimental classroom and observations of the teacher. This implies a caveat concerning a typical classroom, given the presence of research staff (present on 60% of the days), although all classrooms often had adult helpers coming in and out, and the teacher said that children quickly adapted to all the study's components (e.g., note taking and videotaping). On the other hand, observations of the class assured a close evaluation of, and a moderate (site 2) or high (site 1) degree of, implementation fidelity. That is, one or more staff members were easily available and the teacher occasionally asked them questions about the curriculum; also, if any aspect of the implementation was faulty, staff discussed the aspect with the teacher. The moderate implementation at site 2 was due to the availability of about 3 days per week for mathematics and the resulting use of most, but not all, of the curriculum's components. That is, there was little use of "every day" mathematics, such as discussion of mathematics during play. However, this was an optional component and, because all required activities were conducted, the overall implementation was judged to be adequate.

Instrument

The *Building Blocks Assessment of Early Mathematics, PreK–K* (Sarama & Clements, in press) uses an individual interview format, with explicit protocol, coding, and scoring procedures. It assesses children's thinking and learning along research-based developmental progressions for topics of mathematics considered significant for preschoolers, as determined by a consensus of participants in a national conference on early childhood mathematics standards (Clements, Sarama, et al., 2004), rather than mirroring the experimental curriculum's objectives or activities. The assessment was refined in three pilot tests. Content validity was assessed via an expert panel review; concurrent validity was established with a .86 correlation with another instrument (Klein, Starkey, & Wakeley, 2000). The assessment is administered in two sections, each of which takes 20 to 30 minutes per child to complete. The interviews were conducted by doctoral students who had been previously trained and evaluated until they achieved a perfect evaluation on three consecutive administrations. All assessments were videotaped and subsequently coded by independent coders, also previously trained and evaluated. Codes included correct/incorrect evaluations and separate codes for children's strategies in cases where those strategies were intrinsically related to the level of thinking that the item was designed to measure. Coders were naïve as to the treatment group. Results were accumulated and analyzed by an independent professor of educational psychology who is expert in research design and statistics.

The number section measures eight topics, summarized in Table 2. The assessment proceeds along research-based developmental progressions (Clements, Sarama, et al., 2004; see also Figures 1 and 2) for each of these topics until the child makes three consecutive errors. The final items measure skills typically achieved at 7 years of age (Griffin et al., 1995). The maximum score is 97; for this sample, children reached items associated with 6.5 years of age (i.e., all children missed three in a row before reaching items for ages 7–8); therefore, the maximum that these preschoolers could have reached was 72 (coefficient alpha reliability, r = .89; interrater reliability of data coders, 98%).

The geometry test measures seven topics, including measurement and patterning (see Table 2). As with the number section, difficulties for some items on the geometry assessment were designed to measure abilities at 8 years of age. Children complete all 17 items (with several having multiple parts), for a maximum score of 30 (coefficient alpha reliability, r = .71; interrater reliability of data coders, 97%).

Curricula

The *Building Blocks* curriculum has gone through several iterations, the final one (Clements & Sarama, 2007) includes a teacher's edition, including daily whole- and small-group activities and games, free-choice learning centers, and ideas for integrating mathematics throughout the school day; computer software; and books, game sheets, and manipulatives. The *Building Blocks* Pre-K software includes 11 activity sets or scenarios, each including between two and six activities. For example, the

Table 2
Content of the Building Blocks Assessment of Early Mathematics, PreK–K

Topic	No.	Subtopics	Selected examples
		Number	
Verbal Counting	7	Forward Backward Up from a number Before/after/between Identifying mistakes	"How high can you count? Start at 1 and show me."
Object Counting	15	Arrays and scattered arrangements Producing groups Identifying mistakes	"Here are our pennies to pay for food" [8 in a scrambled arrangement]. "Show me how you can count the pennies and tell me how many there are."
Number Recognition and Subitizing	7	Recognizing small groups, untimed Subitizing	Name the number for a group of 2. "I'm going to show you some cards, just for a quick moment! [2 seconds] Try to tell me how many dots on each one."
Number Comparison	19	Nonverbal comparison	[Show cards with ••• and ∴]: "Do these cards have the same number of dots?"
		Verbal comparison	"Which is bigger: 7 or 9?"
Number Sequencing	3	Sequencing	Order cards with 1–5 dots.
Numerals	5	Matching written numerals to quantities	Match numeral and dot cards, 1–5.
Number Composition	6	Number composition and decomposition	"I am putting 5 blocks on this paper? Now, I'm going to hide some." Secretly hide 3. "How many am I hiding?"
Adding and Subtracting	23	Concrete situations Verbal story problems Mental arithmetic	"Pretend I give you 3 candies and then I give you 2 more. How many will you have altogether?"
Place Value	4	Relative size of numbers	Which is closer to 45, 30 or 50?
		Geometry	
Shape Identification	4	Identifying squares, rectangles, triangles, and rhombuses	Given a large array of manipulatable figures (see Figure 3a), choose all exemplars of the stated shape.
Shape Composition	5	Composing and decomposing shapes	Choose the shapes that would result if a shape were cut (see Figure 3b).
Congruence	2	Matching congruent shapes	Given 8 shapes, identify pairs that are "the same shape and same size."
Construction of Shape	2	Building a shape from its components	"Can you make a triangle using some of the straws?"
Turns	1	Recognizing rotation	Analogy (A:B::C:?) with objects rotated 90°.
Measurement	1	Measuring length	"Which of these strings is about the same length as 4 cubes?"
Patterning	1	Copying and extending	Copy, then extend, an ABAB shape pattern.

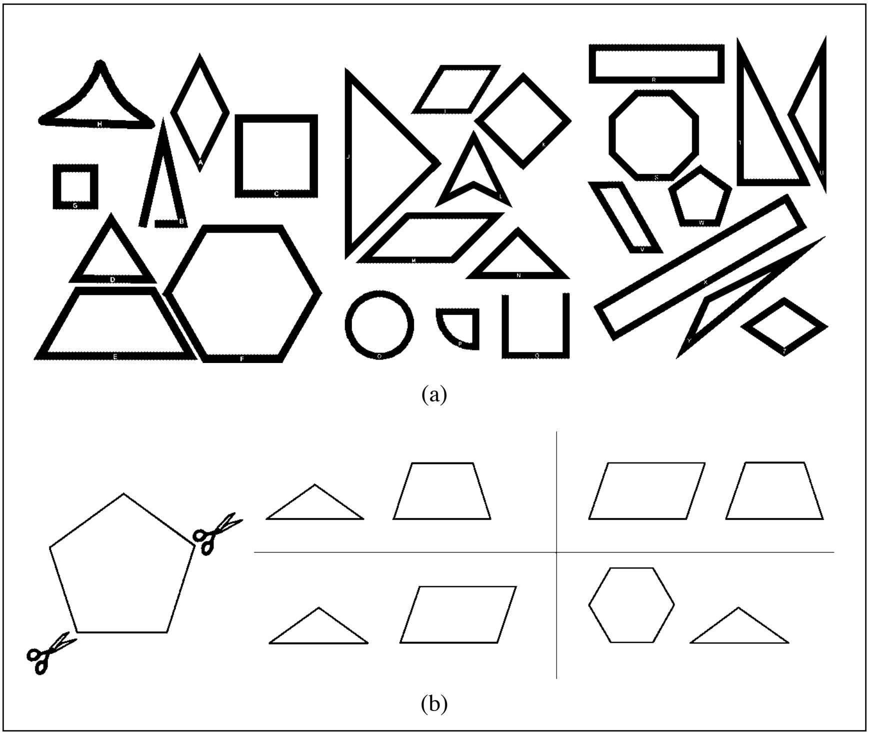


Figure 3. Sample geometry items from the Building Blocks Assessment of Early Mathematics, PreK–K (Sarama & Clements, in press). (a) Given the illustrated cut-out shapes, the child is asked to “Put only the triangles on this paper.” (b) The child is asked, “Pretend you cut this pentagon from one corner to the other. Which shows the two cut pieces?”

“Pizza Pizzazz” scenario includes activities on recognizing and comparing number, counting, and arithmetic. In the first three activities, children match pizzas with the same number of toppings (early number recognition and comparing), create a pizza with the same number of toppings as a given pizza (counting to produce a set), and create a pizza that has a given number of toppings given only a numeral (counting to produce a set that matches a numeral). Later activities in that setting involve addition (see the third column for *Nonverbal Addition* and *Find Change* in Figure 1). The software’s management system presents tasks, contingent on success, along research-based learning trajectories. Activities within various scenarios are introduced according to the trajectory’s sequences. Figure 1 illustrates, for example, how two activities from the “Dinosaur Shop” scenario are sequenced between two illustrated activities from the “Pizza Pizzazz” scenario. Off-computer activities, such as learning center activities, involve corresponding activities. For example, corresponding to the first “Pizza Pizzazz” activity, the teacher sets out a learning center by hiding paper “pizzas” with different numbers of toppings under several opaque containers and placing one such pizza with three toppings in plain view. Children lift each container and count the toppings until they find the matching pizza. They then show the teacher or other adult.

All participating teachers maintained their typical schedule, including circle (whole-group) time, work at centers, snack, outdoor play, and so forth for the 25 school weeks between pretesting and posttesting. The experimental teachers merely inserted the *Building Blocks* activities at the appropriate point of the day. For example, circle time might include a finger play that involved counting and a brief introduction to a new center or game. Center time would include individual work at the curriculum's software or learning centers, guided by the teacher or aide as they circulated throughout the room (specific suggestions for guidance are specified in the curriculum). As a specific example, children might be introduced to new puzzles such as those at the level of the *Picture Maker* level of Figure 2, then engage in physical puzzles with pattern blocks and tangrams in a learning center, or similar puzzles in the "Piece Puzzler" software activity. Teachers guided children by discussing the task, eliciting children's strategies, and, when necessary, modeling successful strategies. Table 3 summarizes the number of activities in which the *Building Blocks* classes engaged, classified by their major goal (many activities addressed multiple goals; activities that were conducted on and off the computer are counted once). The site 1 comparison teacher agreed to continue using her school's mathematics activities, which included number sense (counting with correspondence, numerals, ordinality), operations (uniting sets, counting on, sharing equally), modeling/representing (showing spatial relationships, making charts and graphics, representing number), measurement (comparing, nonstandard measurement, choosing tool, estimation), data (collect and display in graphics and charts) and reasoning (patterns, sorting and classifying, and explaining their actions), and uncertainty (estimation, using spinners, discussing un/certainty). The site 2 compar-

Table 3
Number of Topics Taught in Building Blocks Classrooms by Site

Topic	Site 1	Site 2
Number		
Counting	32	49
Number Recognition and Subitizing	6	6
Number Comparison	9	7
Number Sequencing	4	6
Numerals	8	6
Number Composition	4	4
Adding and Subtracting	13	11
Geometry		
Shape Identification	18	14
Shape Composition	9	9
Congruence	2	2
Construction of Shape	2	1
Turns	1	1
Measurement	1	1
Patterning	4	6

ison teacher used Creative Curriculum (Teaching Strategies Inc., 2001) as well as some "home-grown" curricular activities for mathematics. Visits to those classrooms indicated that each was following the curricula as written.

Analyses

To assess the effectiveness of the curriculum, we conducted factorial repeated measures analyses, with time as the within-group factor, and two between-group factors, school and treatment, evaluating differences in achievement from pre- to posttest on both tests (children did work in the same class, but the software and center activities were engaged in individually, so the child was used as the unit of analysis). In addition, two effect sizes were computed for each test. We compared experimental posttest (E2) to the comparison posttest (C2) scores as an estimate of differential treatment effect. We also compared experimental posttest to experimental pretest (E2 to E1) scores as an estimate of the achievement gain within the experimental curriculum. Effect sizes were computed using adjusted pooled standard deviations (Rosnow & Rosenthal, 1996). We used the accepted benchmarks of .25 or greater as an effect size that has practical significance (i.e., is educationally meaningful), .5 for an effect size of moderate strength, and .8 as a large effect size (Cohen, 1977).

RESULTS

Table 4 presents the raw data for the number and geometry tests. We computed factorial repeated measures analyses, originally including gender; as no main effects or interactions were significant, we present here only the more parsimonious model.

Table 4
Means and Standard Deviations for Number and Geometry Tests by Site and Group

	Building Blocks					
	Site 1		Site 2		Total	
Test	Pre	Post	Pre	Post	Pre	Post
Number	12.38 (10.94)	36.55 (11.12)	6.13 (6.61)	20.17 (13.29)	9.67 (9.70)	29.46 (14.47)
Geometry	9.53 (2.31)	17.69 (2.64)	7.53 (1.86)	12.87 (3.64)	8.79 (2.26)	15.91 (3.81)
	Comparison					
	Site 1		Site 2		Total	
Test	Pre	Post	Pre	Post	Pre	Post
Number	14.07 (9.39)	26.86 (8.64)	3.17 (2.72)	9.56 (9.34)	8.44 (8.69)	17.93 (12.48)
Geometry	9.56 (1.48)	12.12 (2.10)	7.37 (1.40)	8.62 (3.76)	8.63 (1.89)	10.64 (3.35)

Note. These were the data used for the factorial analyses, so they represent data on those children who took all subtests at both the pretest and posttest.

Number

For the number test, there were significant main effects for time (pre-post), $F(1, 57) = 183.19$, $MSE = 33.95$ $p < .0005$; treatment, $F(1, 57) = 6.02$, $MSE = 145.67$, $p < .05$; and site $F(1, 57) = 33.48$, $MSE = 145.67$, $p < .0005$; as well as significant interactions for time by treatment, $F(1, 57) = 20.10$, $MSE = 33.95$, $p < .0005$; and time by site, $F(1, 57) = 15.19$, $MSE = 33.95$, $p < .0005$. Inspection of the means indicates that all groups made significant gains from pretest to posttest; the site 1 scores increased more than those of site 2, and the experimental treatment group score increased more than the comparison group score. The effect size comparing E2 to C2 was .85, and the effect size comparing E2 to E1 was 1.61.

Geometry, Measurement, and Patterns

Likewise for geometry, there were significant main effects for time, $F(1, 49) = 139.08$, $MSE = 3.40$, $p < .0005$; treatment, $F(1, 49) = 17.623$, $MSE = 4.44$, $p < .0005$; and site $F(1, 49) = 27.94$, $MSE = 4.44$, $p < .0005$; as well as significant interactions for time by treatment, $F(1, 49) = 43.57$, $MSE = 3.40$, $p < .0005$; and time by site, $F(1, 49) = 7.95$, $MSE = 3.40$, $p < .01$. Inspection of the means indicates that all groups made significant gains, site 1 scores increased more than those of site 2, and the experimental treatment group score increased more than the comparison group score. The effect size comparing E2 to C2 was 1.47, and the effect size comparing E2 to E1 was 2.26.

Specific Topics

To illuminate which specific topics were affected by the curriculum, Table 5 presents means and standard deviations for the number and geometry subtests. Because we did not wish to increase alpha error and some subtests had a small number of items, we did not perform additional interferential statistics; however, an inspection of the means indicates that the effects were more pronounced on some topics, although positive effects were found for every topic except one. In the realm of number, the smallest relative effects were on object counting and comparing number; for both topics, the experimental group gained more points, but both groups nearly doubled their pretest scores. Both groups made large gains in verbal counting and connecting numerals to groups, with the experimental group's gain the larger. The experimental group's gains were even larger, relative to the comparison group, for the related topics of adding/subtracting and composing number. The largest relative gains in number were achieved in subitizing (tell how many dots are on a card with 5 to 10 dots, shown for 2 seconds) and sequencing (e.g., placing cards with groups of 1 to 5 dots in order from fewest to most).

In geometry, the relative effect on the turn item was small (and higher for the comparison group) and the effect on congruence was positive, but small. Effects on construction of shapes and spatial orientation were large. The largest relative

Table 5
Means and Standard Deviations for Number and Geometry Subtests by Treatment Group

	Building Blocks		Comparison		
Subtest	Pre	Post	Pre	Post	Maximum
		Number			
Verbal Counting	1.13 (1.10)	2.88 (1.51)	0.84 (.93)	1.78 (1.39)	6
Object Counting	5.53 (4.71)	10.97 (4.20)	4.66 (3.89)	8.16 (4.71)	16
Comparing	1.10 (0.99)	2.13 (0.90)	0.89 (0.85)	1.58 (0.96)	5
Numerals	0.60 (1.59)	3.90 (1.86)	0.32 (1.16)	2.48 (2.38)	5
Sequencing	0.07 (0.25)	1.20 (1.24)	0.05 (0.23)	0.39 (0.72)	3
Subitizing	0.18 (0.35)	2.81 (2.63)	0.23 (0.72)	1.00 (1.27)	10
Adding/ Subtracting	0.93 (1.68)	4.20 (2.80)	0.68 (1.38)	2.23 (2.57)	12
Composing	0.13 (0.51)	1.37 (2.31)	0.16 (0.72)	0.32 (0.91)	15
Total	9.67 (9.70)	29.46 (17.95)	7.83 (8.28)	17.93 (12.48)	72
		Geometry, Measurement, Patterning			
Shape Identification	5.42 (0.92)	7.34 (1.16)	5.66 (0.93)	5.89 (1.42)	10
Composition	1.07 (1.10)	4.47 (1.92)	1.23 (1.36)	2.01 (1.65)	11
Congruence	1.02 (0.38)	1.32 (0.35)	1.05 (0.39)	1.20 (0.52)	2
Construction	0.09 (0.23)	0.61 (0.56)	0.09 (0.29)	0.38 (0.48)	2
Orientation	0.15 (0.21)	0.31 (0.28)	0.08 (0.15)	0.08 (0.14)	1
Turns	0.38 (0.49)	0.41 (0.50)	0.21 (0.42)	0.27 (0.45)	1
Measurement	0.10 (0.31)	0.19 (0.40)	0.08 (0.18)	0.08 (0.23)	1
Patterning	0.50 (0.55)	1.26 (0.78)	0.23 (0.42)	0.73 (0.67)	2
Total	8.79 (2.26)	15.91 (3.81)	8.63 (1.89)	10.64 (3.35)	30

Note. These data are from all 68 children; 7 children missed some subtests. Therefore, the average totals differ slightly from those in Table 4 in some cases.

gains in geometry were achieved on shape identification and composition of shapes. Effects on measurement and patterning were moderate.

Children's Strategies

Several items for which the research indicated that a level of sophistication in solution strategies was intrinsically related to the development of each subsequent level of the trajectory (Clements & Sarama, 2004c; Clements, Sarama, et al., 2004; Clements, Wilson, et al., 2004) were coded to describe children's strategies (see Table 6). Results on strategies support the scored results and provide additional

Table 6
Percentage of Children Using Strategies by Treatment Group

	Experimental		Comparison	
	Pre	Post	Pre	Post
Number				
[Show 2 cubes and ask] "How many?"				
Reproduced the set but could not give the number name	13.3	0.0	5.3	16.1
Gave the number	66.7	100.0	65.8	61.3
No response	36.8	0.0	28.9	22.6
[Set out 4 cubes and 5 marbles, cubes physically larger and ask] "Are there more blocks or more marbles or are they the same?"				
Does not match or count (in a reliability observable manner)	16.7	36.6	10.5	22.6
Uses matching	0.0	3.3	0.0	3.2
Counts	6.7	36.7	0.0	16.1
No response	76.7	23.3	89.5	58.1
Counting scrambled arrangements of objects				
Reading order—left to right, top to bottom	3.3	10.0	2.6	16.1
Similar strategy but different directions (e.g., top to bottom)	0.0	40.0	2.6	12.9
Around the perimeter, then moving in to the middle	0.0	10.0	0.0	3.2
Other path through the objects	0.0	3.3	2.6	3.2
No response	96.7	36.7	92.1	64.5
Adding 5 + 3 with objects suggested				
Uses objects	3.3	33.3	2.6	19.4
No objects; solves with verbal counting strategy	3.3	23.3	0.0	6.5
No response	93.3	43.3	97.4	74.2
Solving 3 + 2 with objects nearby but not suggested				
Uses objects	0.0	23.3	0.0	9.7
No objects; solves with verbal counting strategy	0.0	13.3	2.6	6.5
No response	100.0	63.3	97.4	83.9
Solving: 6 dogs and 4 bones, how many dogs wouldn't get a bone? with objects suggested				
Uses objects	3.3	30.0	0.0	12.9
No objects; solves with verbal counting strategy	0.0	3.3	2.6	3.2
No response	96.7	66.7	97.4	83.9

Table 6—*Continued*

	Experimental		Comparison	
	Pre	Post	Pre	Post
Geometry				
Using pattern blocks to fill a puzzle (outline)				
Placing pieces randomly on puzzle that are not connected	79.3	11.1	90.9	76.9
Putting shapes together without leaving gaps	20.7	88.9	9.1	23.1
No response	0.0	0.0	0.0	0.0
Turning shapes after placing them on the puzzle in an attempt to get them to fit	6.9	22.2	6.1	23.1
Turning them into correct orientation prior to placing them	13.8	66.7	3.0	0.0
No response	79.3	11.1	90.9	76.9
Trying out shapes by picking them seemingly at random, then putting them back if they do not look right, so seemingly trial and error	6.9	22.2	6.1	15.4
Appearing to search for "just the right shape" that they "know will fit" and then finding and placing it	13.8	66.7	3.0	7.7
No response	79.3	11.1	90.9	76.9
Hesitant and not systematic	13.8	33.3	6.1	15.4
Overall, solving the puzzle immediately, systematically and confidently	6.9	55.6	3.0	7.7
No response	79.3	11.1	90.9	76.9

Note. Experimental *N*s: pretest, 29; posttest, 27. Comparison *N*s: pretest, 33; posttest 26.

description of the different abilities of the two groups. For the first object counting item, about 66% of both experimental and comparison children could provide a verbal response at pretest. At posttest, 100% of experimental children did so, however 16% of the comparison children reproduced the set but could not give the verbal responses and 23% gave no response. On a number comparison item, experimental children increased their use of a counting strategy more than comparison children, over half of whom did not respond. On items in which children counted scrambled arrangements of objects, the experimental group increased their use of strategies more than the comparison group, especially systematic strategies such as progressing top to bottom, left to right. On the arithmetic items, more children used objects, and fewer used verbal strategies (a small minority on the comparison item, "how many dogs wouldn't get a bone?").

Examining the addition learning trajectory reveals the curriculum's positive effects in more detail. Increases in percentage correct from pretest to posttest for four illustrative items were as follows: 2 + 1 (increase of 37 for experimental vs. 23 for comparison), 3 + 2 (47 vs. 13) 5 + 3 (23 vs. 16), 6 – 4 (how many dogs wouldn't get a bone?—23 vs. 10). The curriculum follows the learning trajectory described in Figure 1. On average, children worked on *Nonverbal Addition* activities 4 times, half on computer (see Figure 1) and half off computer. Teachers modeled nonverbal strategies but also encouraged post hoc verbal reflection. Children worked on *Small Number Addition* activities 6 times, 2 on and 4 off computer (Figure 1). Teachers focused on the meaning of addition as combining two disjoint sets, expressed informally. Children worked on *Find Result* activities 6 times, 2 on and 2 off computer. Use of a child's invented counting strategies to solve join, result unknown problems was emphasized. Finally, children worked on *Find Change* problems 2 times, half on and half off computer. Both on- and off-computer activities emphasized counting on from a given number. The results of these activities is shown in the greater than double increase in correctness by the experimental group, as well as their greater use of solution strategies overall and greater use of more sophisticated strategies, such as verbal counting strategies, for most tasks. For example, on 5 + 3, 33% of the experimental children, compared to 19% of the comparison children, used objects, and 23% of the experimental children, compared to less than 7% of the comparison children, used verbal counting strategies. These results are particularly striking when considering that such tasks are normally part of the first grade curriculum.

Table 6 shows four codes describing children's strategies on a shape composition task. By posttest, experimental children were far more likely to combine shapes without leaving gaps; turn shapes into correct orientation prior to placing them on the puzzle; search for a correct shape; and solve the puzzle immediately, systematically, and confidently. This increased use of more sophisticated shape composition strategies suggests the development of mental imagery.

This development of more sophisticated strategies in the experimental group, along with the large relative gains on the subtest score, more than four times as large as those made by the comparison group (Table 5), substantiate the curriculum's positive effect on geometric composition. The curriculum engages children in several activities to develop this competence, including creating free-form pictures with a variety of shape sets, such as pattern blocks and tangrams, and solving outline puzzles with those same shape sets. Informal work with three-piece foam puzzles and clay cutouts were conducted for several weeks during mid fall. In April, the outline puzzles, which provided the most guidance along the learning trajectories, were introduced. Most children in the present classrooms worked about 2 days on the puzzles designed for children at the *Pre-Composer* level (see the third column in Figure 2), 3 days at the *Piece Assembler* level, and 2 days at the *Picture Maker* level. Only about a third of the children completed all those puzzles, and thus could be confidently classified as operating at the *Shape Composer* level or above. Four children appeared to operate at best at the *Piece Assembler* level. Based on evidence that the developmental sequence of this learning trajectory is valid (Clements, Wilson, et al., 2004), we guided individual children to work on puzzles at the level they had not mastered on the off-computer puzzles. The computer automatically monitored their progress on the "Piece Puzzler" software. The combination of off- and on-computer activities at an appropriate, progressive developmental level appeared to facilitate children's development of the mental actions on objects that engendered thinking at each subsequent level. This is shown in the strategies they employed (Table 6). Almost 90% of the children placed shapes together without leaving gaps, an indication of thinking at the *Picture Maker* level or above. About 67% of the children turned shapes into the correct orientation *prior* to physically placing them within the puzzle outline. The same percentage appeared to search for "just the right shape" that they "knew would fit." These behaviors are criteria for the *Shape Composer* level. Only about 56% however, showed immediate, confident, systematic completion of puzzles. Children's strategies therefore suggest that on the assessment roughly 10% were at the Piece Assembler level, 23% were operating at the Picture Maker level, 11% were in transition to the *Shape Composer* level, and 56% were at the *Shape Composer* level (or above; subsequent levels were not assessed). In contrast, the comparison group had roughly 77% at the *Pre-Composer* or *Piece Assembler* levels, 15% at the *Picture Maker* level, and 8% in transition to the *Shape Composer* level, consistent with developmental averages for this age group (Clements, Wilson, et al., 2004).

DISCUSSION AND IMPLICATIONS

The main purpose of this research was to measure the efficacy of a preschool mathematics program based on a comprehensive model of developing research-based software and print curricula, on a small scale under well-supervised conditions (*Summative Research: Small Scale*). Scores at site 1, the state-funded preschool, increased more than those of site 2, the Head Start school. Site 1 teachers were more experienced than those at site 2. Site 2 children had lower mathematics scores at the start; qualitative observations confirm that site 2 children entered preschool with fewer cognitive resources (attention, metacognition, disposition to learn mathematics, etc.; research reports are under preparation). This is a concern for those involved with Head Start. However, there was no evidence that the curriculum was differentially effective at the two sites. Average achievement gains of the experimental group about doubled those considered large (Cohen, 1977) and approached the sought-after 2-sigma effect of individual tutoring (Bloom, 1984).

Inspection of means for individual topics (subtests) substantiates our conversations with the comparison teachers that they emphasized object counting, comparing numbers, and, to a lesser extent, shapes. The *Building Blocks* curriculum seems to have made a special contribution, with quite large relative gains, to children's learning of the topics of subitizing, sequencing, shape identification, and composition of shapes. Thus, even a moderate number of experiences (e.g., 4 to 6 for sequencing and subitizing) was sufficient to enhance children's learning of certain oft-ignored topics. In addition, the experimental group showed a greater increase in the use of more sophisticated numerical strategies and the development of spatial imagery.

Many have called for more research on the effects of curriculum materials (e.g., Senk & Thompson, 2003), especially because such materials have a large influence on teaching practices (Goodlad, 1984; Grouws & Cebulla, 2000; Woodward & Elliot, 1990). Results of this study indicate strong positive effects of the *Building Blocks* materials, with achievement gains near or approximately equal to those recorded for individual tutoring. This provides support for the efficacy of curricula built on comprehensive research-based principles. The *Building Blocks* materials include research-based computer tools that stand at the base, providing computer analogs to critical mathematical ideas and processes. These are used, or implemented, with activities that guide children through research-based learning trajectories (developed over years of synthesizing our own and others' empirical work). These activities-through-trajectories connect children's informal knowledge to more formal school mathematics. The result is a package that is motivating for children but, unlike "edu-tainment," results in significant assessed learning gains. We believe these features lead to *Building Blocks'* substantial impact, although the present design does not allow attributing the effect to any particular feature or set of features (we are analyzing the qualitative data from these same classrooms that will provide insights relevant to this issue). One practical implication is that, when implemented with at least a moderate degree of fidelity, such materials are highly efficacious in helping preschoolers learn fundamental mathematics concepts and skills.

The results also provide initial, "proof of concept" support for the CRF (Clements, 2007), which extends and particularizes theories of curriculum research (Walker, 1992). Our own use of the CRF emphasizes the actions on objects that should mirror the hypothesized mathematical activity of students and how that activity develops along learning trajectories (Clements, 2002; Clements & Battista, 2000). Such synthesis of curriculum/technology development as a scientific enterprise and mathematics education research may help reduce the separation of research and practice in mathematics and technology education and produce results that are immediately applicable by practitioners (parents, teachers, and teacher educators), administrators and policymakers, and curriculum and software developers. Of course, multiple studies, including comparisons, would need to be conducted to support any claims about the efficacy of the model per se.

Even a small number of experiences for certain topics, such as sequencing number and subitizing, were sufficient to produce large relative learning gains. We believe these topics may often be ignored in most early childhood classrooms. The experimental activities assessed here were efficacious, but other approaches to these topics might also be studied. In contrast, results on such topics as congruence, turn, and measurement indicate that future research should ascertain whether the small number of experiences (1–2) or the nature of the activities dedicated to these topics accounted for the small gains and whether changes to either or both could increase children's achievement.

This study is consistent with extant research showing that organized experiences result in greater mathematics knowledge upon entry into kindergarten (Bowman et al., 2001; Shonkoff & Phillips, 2000) and that focused early mathematical interventions help young children develop a foundation of informal mathematics knowledge (Clements, 1984), especially for children living in poverty (Campbell & Silver, 1999; Fuson, Smith, & Lo Cicero, 1997; Griffin, 2004; Griffin et al., 1995;

Ramey & Ramey, 1998). It extends this research by suggesting that a comprehensive mathematics curriculum following NCTM's standards (2000) can increase knowledge of multiple essential mathematical concepts and skills (beyond number). Unfortunately, most American children are not in high-quality programs (Hinkle, 2000). We recommend that preschool programs adopt research-based curricula (e.g., Clements, 2007). With its emphasis on low-income children, this study also extends the research on standards-based mathematics curricula, most of which does not address social class or cultural influences (cf. Senk & Thompson, 2003).

An overarching caveat is that this study represents phase 9, *Summative Research: Small Scale* (Clements, 2007). As stated, this was justified because it provides an estimate of effect size under close supervision that ensured fidelity of treatment. However, the small number of classrooms, the use of the child as the unit of analysis inside classrooms, the presence of project staff, and our resultant ability to guarantee at least moderate fidelity limits generalizability and internal validity. Further, although teachers at both comparison sites taught their respective mathematics curricula, the emphasis in the programs was clearly on literacy (i.e., time on mathematics activities was not controlled). The results justify the subsequent use of phase 10, *Summative Research: Large Scale*, which we implemented in the 2003–2005 school years. Finally, the quantitative results reported here will be complemented and extended in corresponding studies of the same classrooms involving qualitative case studies of children learning in the context of the curriculum. The focus of these analyses was on the children's development through the learning trajectories for the various mathematical topics.

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