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Illustrative Mathematics®

Algebra 1

Lesson Sampler

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Algebra 1

Overview of McGraw-Hill *Illustrative Mathematics* v

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Illustrative Mathematics is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.

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What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.

Design Principles

Balancing Conceptual Understanding, Procedural Fluency, and Applications

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

Mathematical Practices are the Verbs of Math Class

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

Build on What Students Know

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001) . In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

Good Instruction Starts with Explicit Learning Goals

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities

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Instructional Model

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word "Let's." They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

Lesson Structure

1. INTRODUCE	2. EXPLORE AND DEVELOP
Warm Up Warm Up activities either: ▪ give students an opportunity to strengthen their number sense and procedural fluency. ▪ make deeper connections. ▪ encourage flexible thinking. or: ▪ remind students of a context they have seen before. ▪ get them thinking about where the previous lesson left off. ▪ preview a calculation that will happen in the lesson.	**Classroom Activities** A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class. **Each classroom activity has three phases.** **The Launch** The teacher makes sure that students understand the context and what the problem is asking them to do.

Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

- Modeling Prompts can be thought of as a project or assignment. They are meant to be launched in class by a teacher, but but can be worked on independently or in small groups by students in or out of class.. We built in maximum flexibility for a teacher to implement these in a way that will work for them.
- The purpose of mathematical modeling is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. There should be room to interpret the problem and a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- Modeling with mathematics is not a solitary activity and students should have support from their teacher and classmate while assessments focus on providing feedback that helps students improve their modeling skills.

Student Work Time
Students work individually, with a partner, or in small groups.

Activity Synthesis
The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

National Governors Association Center for Best Practices (2010). Common Core State Standards for Mathematics. NCEE (2013). What Does It Really Mean to Be College and Work Ready? Retrieved November 20, 2017 from http://ncee.org/college-and-work-ready/

3. SYNTHESIZE

Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

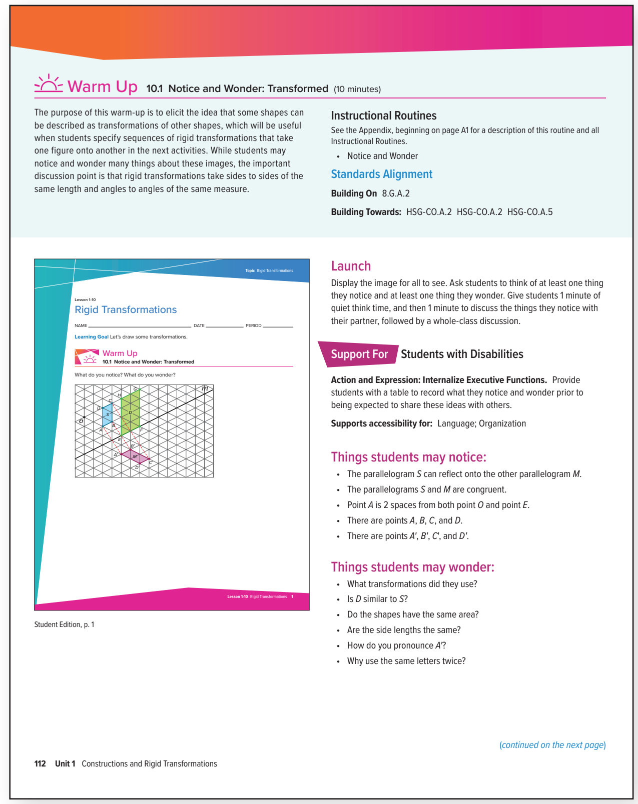
Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These materials include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a well-designed routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Analyze It
- Anticipate, Monitor, Select, Sequence, Connect
- Aspects of Mathematical Modeling
- Card Sort
- Construct It
- Draw It
- Estimation
- Extend It
- Fit It
- Graph It
- Math Talk
- Notice and Wonder
- Poll the Class
- Take Turns
- Think Pair Share
- Which One Doesn't Belong?



Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm-up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

Standards Alignment

Building On 8.G.A.2

Building Towards: HSG-CO.A.2 HSG-CO.A.2 HSG-CO.A.5

Topic Rigid Transformations

Lesson 1-10

Rigid Transformations

NAME ____ DATE ____ PERIOD ____

Learning Goal Let's draw some transformations.

Warm Up

10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

Lesson 1-10 Rigid Transformations 1

Student Edition, p. 1

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Support For Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

Supports accessibility for: Language; Organization

Things students may notice:

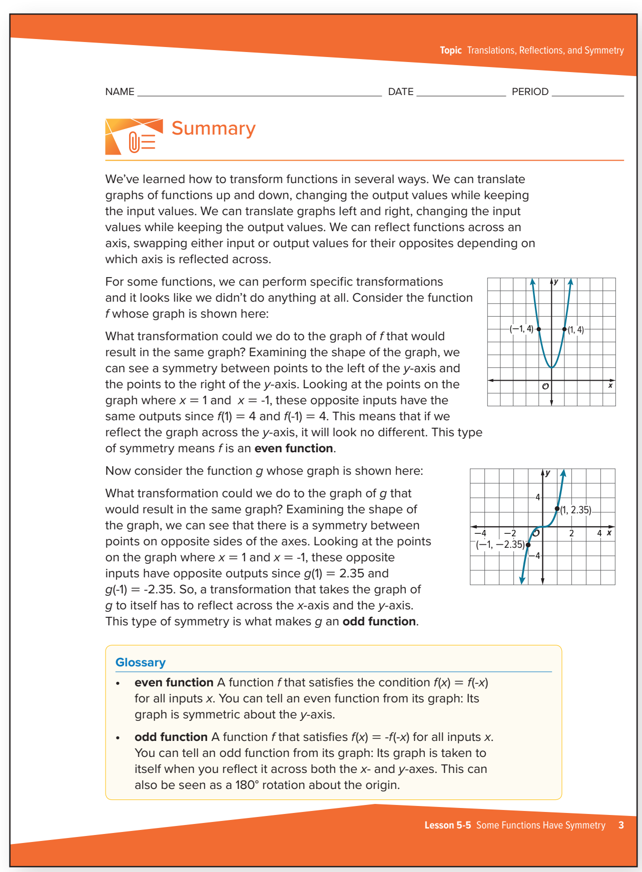
- The parallelogram S can reflect onto the other parallelogram M.
- The parallelograms S and M are congruent.
- Point A is 2 spaces from both point O and point E.
- There are points A, B, C, and D.
- There are points A', B', C', and D'.

Things students may wonder:

- What transformations did they use?
- Is D similar to S?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce A'?
- Why use the same letters twice?

(continued on the next page)

112 Unit 1 Constructions and Rigid Transformations



Topic Translations, Reflections, and Symmetry

NAME ____ DATE ____ PERIOD ____

Summary

We've learned how to transform functions in several ways. We can translate graphs of functions up and down, changing the output values while keeping the input values. We can translate graphs left and right, changing the input values while keeping the output values. We can reflect functions across an axis, swapping either input or output values for their opposites depending on which axis is reflected across.

For some functions, we can perform specific transformations and it looks like we didn't do anything at all. Consider the function f whose graph is shown here:

What transformation could we do to the graph of f that would result in the same graph? Examining the shape of the graph, we can see a symmetry between points to the left of the y-axis and the points to the right of the y-axis. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have the same outputs since $f(1) = 4$ and $f(-1) = 4$. This means that if we reflect the graph across the y-axis, it will look no different. This type of symmetry means f is an **even function**.

Now consider the function g whose graph is shown here:

What transformation could we do to the graph of g that would result in the same graph? Examining the shape of the graph, we can see that there is a symmetry between points on opposite sides of the axes. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have opposite outputs since $g(1) = 2.35$ and $g(-1) = -2.35$. So, a transformation that takes the graph of g to itself has to reflect across the x-axis and the y-axis. This type of symmetry is what makes g an **odd function**.

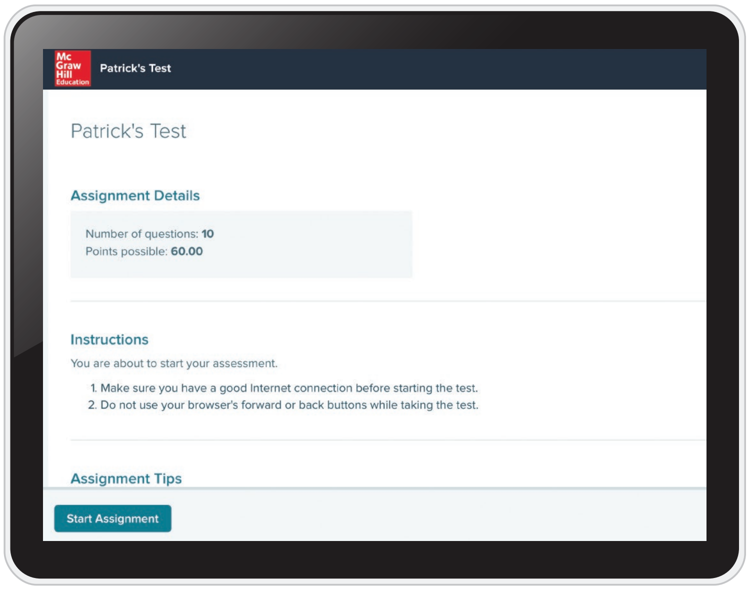
Glossary

- **even function** A function f that satisfies the condition $f(x) = f(-x)$ for all inputs x. You can tell an even function from its graph: Its graph is symmetric about the y-axis.
- **odd function** A function f that satisfies $f(x) = -f(-x)$ for all inputs x. You can tell an odd function from its graph: Its graph is taken to itself when you reflect it across both the x- and y-axes. This can also be seen as a 180° rotation about the origin.

Lesson 5-5 Some Functions Have Symmetry 3

How to Assess Progress

Illustrative Mathematics contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.



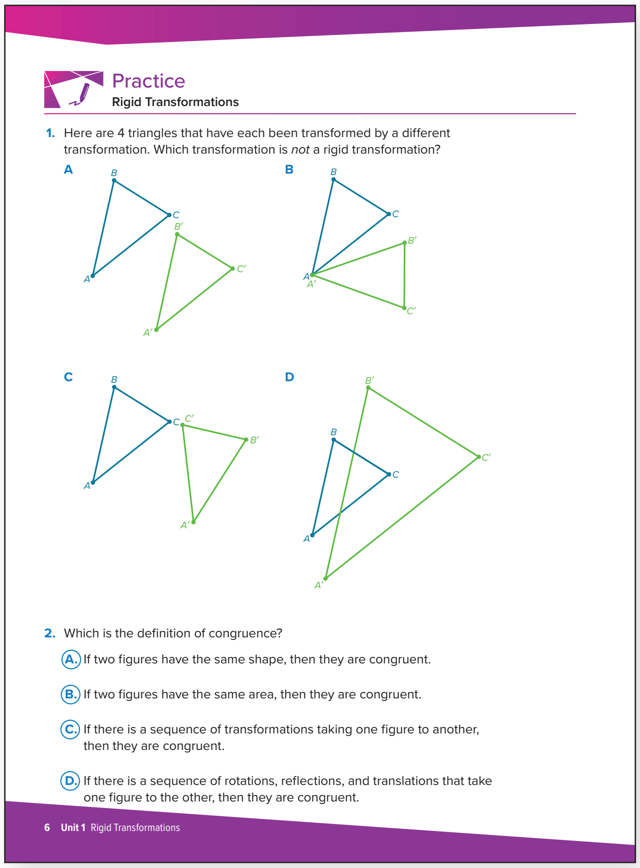
- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.

- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.

- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.

- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.

- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.



Practice
Rigid Transformations

1. Here are 4 triangles that have each been transformed by a different transformation. Which transformation is *not* a rigid transformation?

2. Which is the definition of congruence?

A. If two figures have the same shape, then they are congruent.

B. If two figures have the same area, then they are congruent.

C. If there is a sequence of transformations taking one figure to another, then they are congruent.

D. If there is a sequence of rotations, reflections, and translations that take one figure to the other, then they are congruent.

6 Unit 1 Rigid Transformations

Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Supporting English-language Learners

Illustrative Mathematics builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

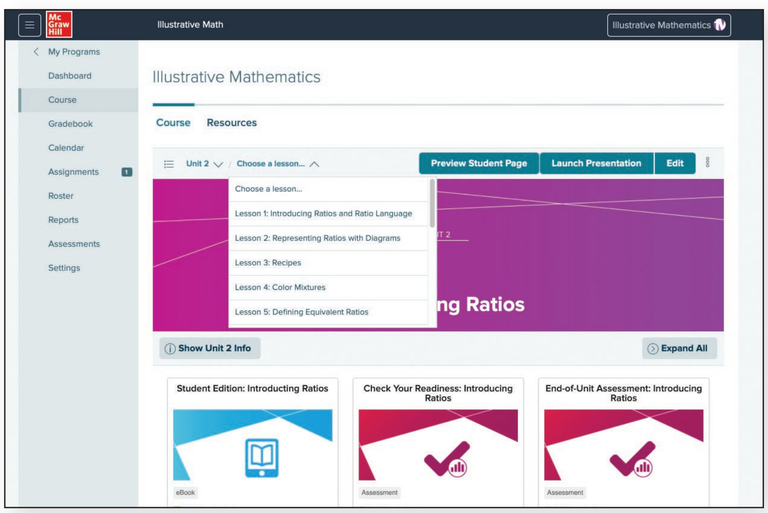
Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

Digital

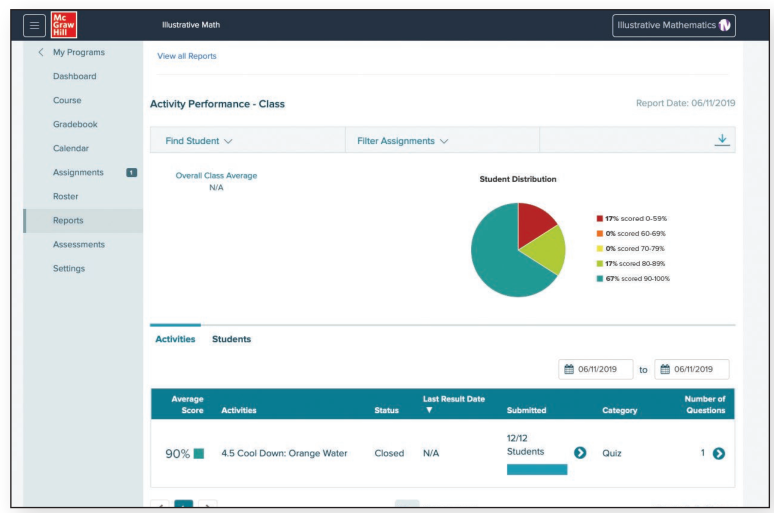
McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

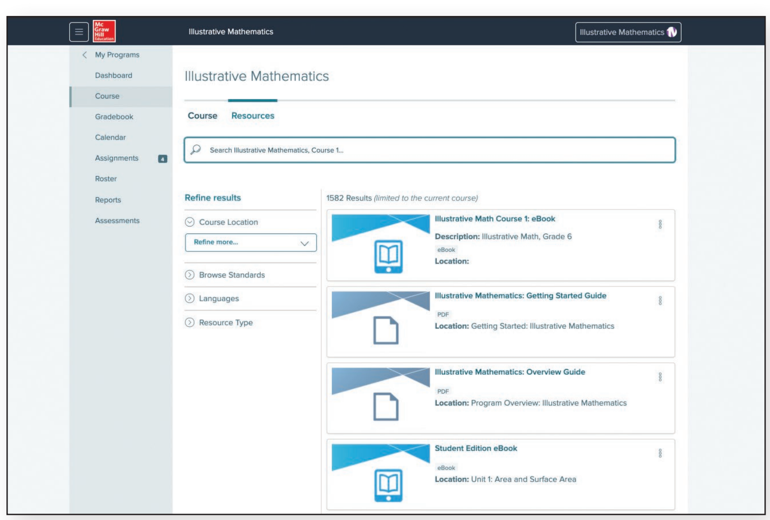
- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting



Launch Presentations Digital versions of lessons to present content.



Reports Review the performance of individual students, classrooms, and grade levels.



Access Resources Point-of-use access to resources such as assessments, eBooks, and course guides.

Unit 1

One-variable Statistics

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Getting to Know You

Lesson 1-1 Getting to Know You

1-2 Data Representations

1-3 A Gallery of Data

Distribution Shapes

1-4 The Shape of Distributions

1-5 Calculating Measures of Center and Variability

How to Use Spreadsheets

1-6 Mystery Computations

1-7 Spreadsheet Computations

1-8 Spreadsheet Shortcuts

Manipulating Data

1-9 Technological Graphing

1-10 The Effect of Extremes

1-11 Comparing and Contrasting Data Distributions

1-12 Standard Deviation

1-13 More Standard Deviation

1-14 Outliers

1-15 Comparing Data Sets

Analyzing Data

1-16 Analyzing Data

Unit 2

Linear Equations, Inequalities and Systems

Syda Productions/Shutterstock

Writing and Modeling with Equations

Lesson 2-1 Planning a Pizza Party

2-2 Writing Equations to Model Relationships (Part 1)

2-3 Writing Equations to Model Relationships (Part 2)

2-4 Equations and Their Solutions

2-5 Equations and Their Graphs

Manipulating Equations and Understanding Their Structure

2-6 Equivalent Equations

2-7 Explaining Steps for Rewriting Equations

2-8 Which Variable to Solve for? (Part 1)

2-9 Which Variable to Solve for? (Part 2)

2-10 Connecting Equations to Graphs (Part 1)

2-11 Connecting Equations to Graphs (Part 2)

Systems of Linear Equations in Two Variables

2-12 Writing and Graphing Systems of Linear Equations

2-13 Solving Systems by Substitution

2-14 Solving Systems by Elimination (Part 1)

2-15 Solving Systems by Elimination (Part 2)

2-16 Solving Systems by Elimination (Part 3)

2-17 Systems of Linear Equations and Their Solutions

Linear Inequalities in One Variable

2-18 Representing Situations with Inequalities

2-19 Solutions to Inequalities in One Variable

2-20 Writing and Solving Inequalities in One Variable

Linear Inequalities in Two Variables

2-21 Graphing Linear Inequalities in Two Variables (Part 1)

2-22 Graphing Linear Inequalities in Two Variables (Part 2)

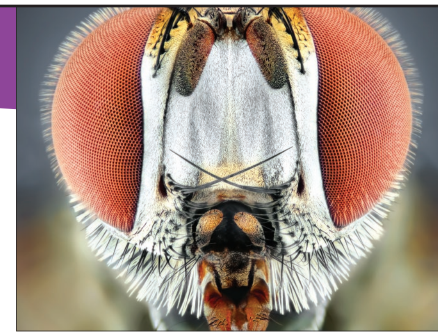
2-23 Solving Problems with Inequalities in Two Variables

Systems of Linear Inequalities in Two Variables

2-24 Solutions to Systems of Linear Inequalities in Two Variables

2-25 Solving Problems with Systems of Linear Inequalities in Two Variables

2-26 Modeling with Systems of Inequalities in Two Variables



Unit 3

Two-Variable Statistics

Two-way Tables

Lesson 3-1 Two-way Tables

3-2 Relative Frequency Tables

3-3 Associations in Categorical Data

Scatterplots

3-4 Linear Models

3-5 Fitting Lines

3-6 Residuals

Correlation Coefficients

3-7 The Correlation Coefficient

3-8 Using the Correlation Coefficient

3-9 Causal Relationships

Estimating Lengths

3-10 Fossils and Flags

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Unit 4

Functions

Functions and Their Representations

Lesson 4-1 Describing and Graphing Situations

4-2 Function Notation

4-3 Interpreting & Using Function Notation

4-4 Using Function Notation to Describe Rules (Part 1)

4-5 Using Function Notation to Describe Rules (Part 2)

Analyzing and Creating Graphs of Functions

4-6 Features of Graphs

4-7 Using Graphs to Find Average Rate of Change

4-8 Interpreting and Creating Graphs

4-9 Comparing Graphs

A Closer Look at Inputs and Outputs

4-10 Domain and Range (Part 1)

4-11 Domain and Range (Part 2)

4-12 Piecewise Functions

4-13 Absolute Value Functions (Part 1)

4-14 Absolute Value Functions (Part 2)

Inverse Functions

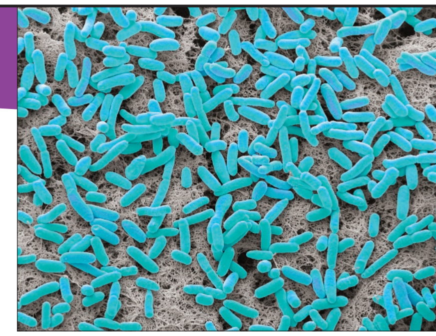
4-15 Inverse Functions

4-16 Finding and Interpreting Inverse Functions

4-17 Writing Inverse Functions to Solve Problems

Putting it All Together

4-18 Using Functions to Model Battery Power



STEVE GSCHMEISSNER/SCIENCE PHOTO LIBRARY/Getty Images

Unit 5

Introduction to Exponential Functions

Looking at Growth

Lesson 5-1 Growing and Growing

5-2 Patterns of Growth

A New Kind of Relationship

5-3 Representing Exponential Growth

5-4 Understanding Decay

5-5 Representing Exponential Decay

5-6 Analyzing Graphs

5-7 Using Negative Exponents

Exponential Functions

5-8 Exponential Situations as Functions

5-9 Interpreting Exponential Functions

5-10 Looking at Rates of Change

5-11 Modeling Exponential Behavior

5-12 Reasoning about Exponential Graphs (Part 1)

5-13 Reasoning about Exponential Graphs (Part 2)

Percent Growth and Decay

5-14 Recalling Percent Change

5-15 Functions Involving Percent Change

5-16 Compounding Interest

5-17 Different Compounding Intervals

5-18 Expressed in Different Ways

Comparing Linear and Exponential Functions

5-19 Which One Changes Faster?

5-20 Changes over Equal Intervals

Putting It All Together

5-21 Predicting Populations

Unit 6

Introduction to Quadratic Functions

©TongRo Image Stock/Alamy Stock Photo

A Different Kind of Change

Lesson 6-1 A Different Kind of Change

6-2 How Does it Change?

Quadratic Functions

6-3 Building Quadratic Functions from Geometric Patterns

6-4 Comparing Quadratic and Exponential Functions

6-5 Building Quadratic Functions to Describe Situations (Part 1)

6-6 Building Quadratic Functions to Describe Situations (Part 2)

6-7 Building Quadratic Functions to Describe Situations (Part 3)

Working with Quadratic Expressions

6-8 Equivalent Quadratic Expressions

6-9 Standard Form and Factored Form

6-10 Graphs of Functions in Standard and Factored Forms

Features of Graphs of Quadratic Functions

6-11 Graphing from the Factored Form

6-12 Graphing the Standard Form (Part 1)

6-13 Graphing the Standard Form (Part 2)

6-14 Graphs That Represent Situations

6-15 Vertex Form

6-16 Graphing from the Vertex Form

6-17 Changing the Vertex

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Unit 7

Quadratic Functions

Finding Unknown Inputs

Lesson 7-1 Finding Unknown Inputs

7-2 When and Why Do We Write Quadratic Equations?

Solving Quadratic Equations

7-3 Solving Quadratic Equations by Reasoning

7-4 Solving Quadratic Equations with the Zero Product Property

7-5 How Many Solutions?

7-6 Rewriting Quadratic Expressions in Factored Form (Part 1)

7-7 Rewriting Quadratic Expressions in Factored Form (Part 2)

7-8 Rewriting Quadratic Expressions in Factored Form (Part 3)

7-9 Solving Quadratic Equations by Using Factored Form

7-10 Rewriting Quadratic Expressions in Factored Form (Part 4)

Completing the Square

7-11 What are Perfect Squares?

7-12 Completing the Square (Part 1)

7-13 Completing the Square (Part 2)

7-14 Completing the Square (Part 3)

7-15 Quadratic Equations with Irrational Solutions

The Quadratic Formula

7-16 The Quadratic Formula

7-17 Applying the Quadratic Formula (Part 1)

7-18 Applying the Quadratic Formula (Part 2)

7-19 Deriving the Quadratic Formula

7-20 Rational and Irrational Solutions

7-21 Sums and Products of Rational and Irrational Numbers

Unit 6

Introduction to Quadratic Functions

Prior Work

Linear, Exponential, and Quadratic Patterns

Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In this unit, they begin by looking at some patterns that grow quadratically. They contrast this growth with linear and exponential growth. They further observe that eventually these quadratic patterns grow more quickly than linear patterns but more slowly than exponential patterns.

Work in This Unit

Standard, Factorial, and Vertex Forms of Quadratic Equations

Students examine the important example of free-falling objects whose height over time can be modeled with quadratic functions. They use tables, graphs, and equations to describe the movement of these objects, eventually looking at the situation where a projectile is launched upward. This leads to the important interpretation that in a quadratic function such as $f(t) = 5 + 30t - 16t^2$, representing the vertical position of an object after t seconds, 5 represents the initial height of the object, $30t$ represents its initial upward path, and $-16t^2$ represents the effect of gravity. Through this investigation, students also begin to appreciate how the different coefficients in a quadratic function influence the shape of the graph. In addition to projectile motion, students examine other situations represented by quadratic functions including area and revenue.

Next, students examine the standard and factored forms of quadratic expressions. They investigate how each form is useful for understanding the graph of the function defined by these equivalent forms. The factored form is helpful for finding when the quadratic function takes the value 0 to obtain the x-intercept(s) of its graph, while the constant term in the standard form shows the y-intercept. Students also find that the factored form is useful for finding the vertex of the graph because its x-coordinate is halfway between the points where the graph intersects the x-axis (if it has two x-intercepts). As for the standard form, students investigate the coefficients of the quadratic and linear terms further, noticing that the coefficient of the quadratic term determines if it opens upward or downward. The effect of the coefficient of the linear term is somewhat mysterious and more complicated. Students explore how it shifts the graph both vertically and horizontally in an optional lesson.

Finally, students investigate the vertex form of a quadratic function and understand how the parameters in the vertex form influence the graph. They learn how to determine the vertex of the graph from the vertex form of the function. They also begin to relate the different parameters in the vertex form to the general ideas of horizontal and vertical translation and vertical stretch, ideas which will be investigated further in a later course.

Note on Materials

Access to graphing technology is necessary for many activities. Examples of graphing technology are: a handheld graphing calculator, a computer with a graphing calculator application installed. For students using the digital version of these materials, a separate graphing calculator tool isn't necessary. Interactive applets are embedded throughout, and a graphing calculator tool is accessible in the student math tools.

Lessons		Days	Standards
Check Your Readiness Assessment		1	6.EE.A.3, 6.EE.B.6, 6.G.A, 6.EE.A.3, 7.EE.A.1, 7.NS.A.1.c, 8.F.A.3, HSA-REI.D.10, HSF-IF.B.6, HSF-BF.A.1.a, 8.EE.B.5, 8.EE.B.6, HSF-BF.B.3, HSF-IF.C.7.a
Topic **A Different Kind of Change**			
Lesson 6-1*	A Different Kind of Change	1	HSF-LE.A.1, HSF-BF.A.1.a, HSF-LE.A,
Lesson 6-2	How Does it Change?	1	HSA-SSE.A.1, HSA-SSE.B.3, HSF-BF.A.1.a
Topic **Quadratic Functions**			
Lesson 6-3*	Building Quadratic Functions from Geometric Patterns	1	HSA-CED.A.2, HSA-SSE.A.1, HSF-LE.A.1, HSA-SSE.A.1, HSF-BF.A.1.a, HSF-IF.A.2
Lesson 6-4	Comparing Quadratic and Exponential Functions	1	6.EE.A.1, HSF-BF.A.1.a, HSF-IF.C, HSF-LE.A.3, HSF-LE.A.3
Lesson 6-5	Building Quadratic Functions to Describe Situations (Part 1)	1	HSF-BF.A.1.a, HSF-IF.A.2, HSF-BF.A.1
Lesson 6-6	Building Quadratic Functions to Describe Situations (Part 2)	1	HSF-BF.A.1, HSF-BF.A.1.a, HSF-IF.B.5
Lesson 6-7	Building Quadratic Functions to Describe Situations (Part 3)	1	HSF-IF.B.4, HSF-BF.A.1.a, HSF-IF.B.5, HSF-IF.C.7.c
Topic **Working with Quadratic Expressions**			
Lesson 6-8	Equivalent Quadratic Expressions	1	6.EE.A.3, 7.EE.A.1, HSA-SSE.A, HSA-SSE.A.2, HSA-SSE.B.3, HSF-IF.C.8
Lesson 6-9	Standard Form and Factored Form	1	HSA-SSE.A.2, HSA-SSE.B.3, HSF-IF.C.8
Lesson 6-10	Graphs of Functions in Standard and Factored Forms	1	HSF-IF.B.4, HSA-SSE.B.3, HSA-SSE.B.3
Mid-Unit Assessment		1	HSF-BF.A.1.a, HSF-LE.A, HSF-IF.B.4, HSF-IF.B.5, HSF-IF.C, HSA-SSE.A.2, HSF-BF.A.1.a, HSF-IF.B.4, HSF-IF.C, HSF-LE.A.3, HSA-SSE.A.2, HSF-IF.B.4, HSF-IF.B.5, HSF-IF.C.7.a
Topic **Features of Graphs of Quadratic Functions**			
Lesson 6-11	Graphing from the Factored Form	1	HSA-SSE.A, HSF-IF.C.7.a, HSF-IF.C.7.c
Lesson 6-12	Graphing the Standard Form (Part 1)	1	HSF-BF.B.3, HSF-IF.C, HSF-IF.C.7, HSF-LE.A.2
Lesson 6-13*	Graphing the Standard Form (Part 2)	1	HSA-SSE.B.3, HSF-BF.B.3, HSF-IF.C.7, HSF-IF.C.7.a
Lesson 6-14*	Graphs That Represent Situations	1	HSF-IF.A.2, HSF-IF.C.7.a, HSF-IF.C.8, HSF-IF.C.9
Lesson 6-15	Vertex Form	1	HSF-BF.B.3, HSF-IF.C, HSF-IF.C.7.a
Lesson 6-16	Graphing from the Vertex Form	1	HSF-IF.C, HSF-IF.C.7.a
Lesson 6-17*	Changing the Vertex	1	HSF-BF.B.3, HSF-IF.C, HSF-IF.C.7.a
End-of-Unit Assessment		1	HSF-IF.C.7, HSA-SSE.A.2, HSF-IF.C.8, HSF-IF.C.7, HSF-IF.C.7.a, HSA-SSE.B.3, HSF-IF.C.7.a, HSF-IF.B.4, HSF-IF.C.9, HSF-BF.B.3, HSF-IF.C.7.a, HSF-IF.C.8, HSF-IF.B.4, HSF-IF.B.5, HSF-IF.C.8
**indicates Lessons in which there are optional activities*	TOTAL	19-20	

Required Materials

- Colored pencils (Lesson 11)
- Graphing technology (Lessons 4, 6, 11, 12, 13, 14, 15, 17)
- Graph paper (Lesson 1)
- Pre-printed cards, cut from copies of the blackline master (Lessons 12, 14, 16)
- Scientific calculators (Lesson 14)

Blackline Activity Masters

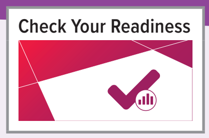
- Card Sort: Representations of Quadratic Functions (Lesson 12, Activity 4)
- Info Gap: Rocket Math (Lesson 14, Activity 4)
- Matching Equations with Graphs (Lesson 16, Activity 3)

Pre-Unit Diagnostic Assessment

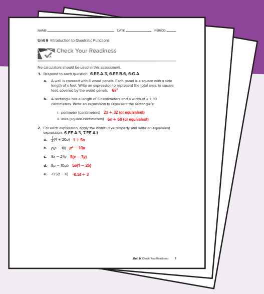
The pre-unit diagnostic assessment, *Check Your Readiness*, evaluate students' proficiency with prerequisite concepts and skills that they need to be successful in the unit. The item descriptions below offer guidance for students who may answer items incorrectly.

The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

Materials Neither scientific nor graphing calculators should be used in this assessment.



Available as a digital assessment or a printable assessment.



1. Item Description

In this unit, students explore quadratic expressions that arise in the context of area calculations. This item assesses a middle school skill: writing expressions to represent area and perimeter when they are given a variable side length.

First Appearance of Skill or Concept: Lesson 3

If most students struggle with this item... .

- Plan to discuss the connection between finding area given a numerical value and given an algebraic expression in Lesson 3 Activity 1.

2. Item Description

Throughout the unit, students write expressions in different forms by applying properties of operations, especially the distributive property.

First Appearance of Skill or Concept: Lesson 8

If most students do well with this item... .

- It may be possible to skip Lesson 8 Activity 2.

NAME ______________ DATE ______ PERIOD ____

Unit 6 Introduction to Quadratic Functions

Check Your Readiness

No calculators should be used in this assessment.

1. Respond to each question. **6.EE.A.3, 6.EE.B.6, 6.G.A**

a. A wall is covered with 6 wood panels. Each panel is a square with a side length of x feet. Write an expression to represent the total area, in square feet, covered by the wood panels. $6x^2$

b. A rectangle has a length of 6 centimeters and a width of $x + 10$ centimeters. Write an expression to represent the rectangle's:

i. perimeter (centimeters) **$2x + 32$ (or equivalent)**

ii. area (square centimeters) **$6x + 60$ (or equivalent)**

2. For each expression, apply the distributive property and write an equivalent expression. **6.EE.A.3, 7.EE.A.1**

a. $\frac{1}{4}(4 + 20a)$ $1 + 5a$

b. $p(p - 10)$ $p^2 - 10p$

c. $8x - 24y$ $8(x - 3y)$

d. $5a - 10ab$ $5a(1 - 2b)$

e. $-0.5(t - 6)$ $-0.5t + 3$

Unit 6 Check Your Readiness 1

3. Item Description

In the unit, students will write equivalent expressions. When rewriting quadratic expressions from factored form to standard form, students will use their understanding that subtracting by a number *n* is the same as adding the opposite of *n*.

First Appearance of Skill or Concept: Lesson 9

If most students do well with this item... .

- Plan to use only one or two questions in Lesson 9 Activity 1 and condense the Lesson 9 Activity 2 Synthesis since students are likely to recognize how to apply the distributive property to the product of two differences or the product of a sum and difference without too much difficulty.

4. Item Description

Students analyze and articulate the connections between a linear equation in two variables and the graph that represents it.

First Appearance of Skill or Concept: Lesson 10

If most students struggle with this item... .

- Plan to spend additional time connecting the parts of equations with the features of the graph starting in Lesson 10 Activity 1.
- Later in this lesson and other lessons, students will make similar connections between graphs of quadratic functions and the equations that define them.

NAME ______________________ DATE ____________ PERIOD ______

3. Which expression is equal to 815−97? **7.NS.A.1.c**

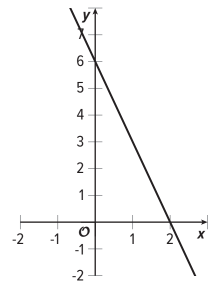
A. 815 − (-97)

B. 815 + 97

C. 815 + (-97)

D. 97 − 815

4. The graph represents the equation $y = -3x + 6$. **8.F.A.3, HSA-REI.D.10**



a. Explain how you can use the equation to tell that the point (5, -9) is on the graph.

Sample response: If we substitute 5 for *x* and -9 for *y* in the equation, we have $-9 = -3(5) + 6$ or $-9 = -15 + 6$, which is a true statement.

b. What is the *y*-intercept of the graph? How is the *y*-intercept related to the equation?

Sample response: The *y*-intercept is (0, 6). If we substitute 0 for the *x* in the equation and evaluate the expression, we get the *y*-coordinate of the *y*-intercept. The constant term in the equation is the *y*-coordinate of the *y*-intercept.

c. What is the *x*-intercept of the graph? How is the *x*-intercept related to the equation?

Sample response: The *x*-intercept is (2, 0). If we substitute 0 for the *y* in the equation and solve the equation, the solution tells us the *x*-coordinate of the *x*-intercept.

2 **Unit 6** Check Your Readiness

5. Item Description

Students read and interpret a continuous graph representing a function, including finding x-intercepts, y-intercept, and the maximum value achieved by the function. The function here is quadratic but is not labeled as such and is presented graphically. (Students have seen graphs of quadratic functions in an earlier unit, though at the time the functions weren't labeled with the term "quadratic.")

First Appearance of Skill or Concept: Lesson 6

If most students struggle with this item... .

- Plan to spend additional time in Lesson 7 and again in Section D interpreting the intercepts and vertex of the graph of a quadratic function in terms of the situation it represents.
- Consider revisiting this question or inserting an additional question that shows projectile motion, and have student interpret the meaning of the vertex and intercepts.

6. Item Description

Students analyze a description of a situation involving a constant rate of change, find output values given some input values, and write linear expressions to represent relationships between quantities.

First Appearance of Skill or Concept: Lesson 6

If most students struggle with this item... .

- Plan to allow additional time for students to write the linear equation in Lesson 6 Activity 1.
- Emphasize connections between the table of values and the parameters in the equation.

NAME ______________ DATE ________ PERIOD ____

5. Andre is standing on a beach, and throws a rock up in the air so that it will land in the ocean.

The graph shows the height of the rock, in feet, above the water as a function of time, in seconds. **HSF-IF.B.6**

a. How high above the water was the rock when Andre threw it? Explain how you know.

About 5 feet. This is where the graph intersects the *y*-axis.

b. When did the rock reach its maximum height? How high was it?

After 1 second, the rock reached its maximum height of 21 feet.

c. About when did the rock hit the ocean? **The rock hit the ocean a little more than 2 seconds after Andre threw it.**

6. A train is traveling at a constant speed of 44 feet per second. At a checkpoint along the route of the train, a sensor detects when the front of the train passes the checkpoint. At that moment, a stopwatch is started. **HSF-BF.A.1.a**

a. Complete the table with the distance between the checkpoint and the front of the train at each time on the stopwatch.

Time on the Stopwatch (Seconds)	Distance from Checkpoint (Feet)
0	**0**
1	**44**
2	**88**
3	**132**
4	**176**

b. Write an expression to represent the distance (feet) of the front of the train from the checkpoint as a function of t, the time (seconds) on the stopwatch. **$44t$**

c. Station Q is 5,000 feet past the checkpoint. How far from Station Q is the front of the train when the stopwatch shows 4 seconds? What about when the stopwatch shows 15 seconds? **The train is 4,824 feet from Station Q when the stopwatch shows 4 seconds and 4,340 feet when the stopwatch shows 15 seconds.**

d. Write an expression to represent the distance (feet) of the front of the train from Station Q as a function of t, the time (seconds) on the stopwatch.
$5{,}000 - 44t$

Unit 6 Check Your Readiness 3

7. Item Description

In this unit, students will investigate how changes to the parameters of a quadratic expression affect the graph that represents it. This item gauges students' understanding on how the graph of a linear function changes when a constant term is added to the expression defining the function.

First Appearance of Skill or Concept: Lesson 12

If most students struggle with this item... .

- Plan to spend time on the optional activity included in Lesson 12 to further reinforce how changing the parameters in an equation changes the outputs and the resulting graph.

8. Item Description

In this unit, students encounter different forms of quadratic expressions. Each form reveals different information about the quadratic functions they represent. This item assesses students' understanding of the structures of linear expressions and the information they reveal.

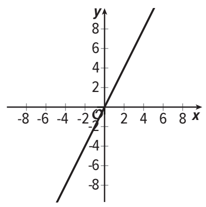
First Appearance of Skill or Concept: Lesson 7

If most students struggle with this item... .

- Plan to spend additional time in this lesson and later lessons when students have to make connections between expressions and real-world situations by making explicit connections between the different representations and making use of suggested math language routines.

NAME ______________________ DATE ____________ PERIOD ______

7. Here is the graph of the equation $y = 2x$.



How would the graph of $y = 2x + 8$ look different from this graph? **8.EE.B.5, 8.EE.B.6, HSF-BF.B.3, HSF-IF.C.7.a**

A. It would have a steeper slope.

B. It would no longer be a line. It would curve upward.

C. It would be shifted 8 units to the right.

D. It would be shifted 8 units up.

8. Han bought a $15 bus pass. Each bus ride costs $1.50. **7.EE.A.2**

a. The expression $15 - 1.50r$ represents the dollar amount left in Han's bus pass after r rides. Explain what the 15, -1.50, and $-1.50r$ in the expression mean in this situation.

In $15 - 1.50r$, the 15 is the initial amount of money in the pass. The -1.50 shows the dollar amount being subtracted for each ride. The $-1.50r$ shows the dollar amount being subtracted after r rides.

b. Show that the expression $1.50(10 - r)$ is equivalent to $15 - 1.50r$.

Distributing 1.50 over $(10 - r)$ gives $(1.50) \cdot 10 - 1.50 \cdot r$, which is $15 - 1.50r$.

c. Explain what the 1.50, 10, and $(10 - r)$ in the expression $1.50(10 - r)$ mean in this situation.

In $1.50(10 - r)$, the 1.50 is the dollar cost per ride. The 10 shows how many rides Han can take with the $15 starting amount. The $(10 - r)$ is the number of rides left after he takes r rides.

4 **Unit 6** Check Your Readiness

Lesson 6-6

Building Quadratic Functions to Describe Situations (Part 2)

Goals (Teacher-Facing)

- Create graphs of quadratic functions that represent a physical phenomenon and determine an appropriate domain when graphing.
- Identify and interpret (orally and in writing) the meaning of the vertex of a graph and the zeros of a function represented in tables and graphs.
- Write and interpret (orally and in writing) quadratic functions that represent a physical phenomenon.

Student Learning Goals

Let's look at the objects being launched in the air.

Learning Targets

- I can create quadratic functions and graphs that represent situations.
- I can relate the vertex of a graph and the zeros of a function to a situation.
- I know that the domain of a function can depend on the situation it represents.

Required Materials

- Graphing technology

Required Preparation

Acquire devices that can run graphing technology. It is ideal if each student has their own device.

Lesson Pacing

	Pacing (min)
Warm Up 6.1 Sky Bound	5
Activity 6.2 Tracking a Cannonball	15
Activity 6.3 Graphing Another Cannonball	15
Lesson Synthesis	5
Cool Down 6.4 Rocket in the Air	5
TOTAL	**45**

Standards Alignment

Addressing

HSF-BF.A.1 Write a function that describes a relationship between two quantities.

HSF-BF.A.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context.

HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

HSF-IF.C Analyze functions using different representations.

HSF-IF.C.7.a Graph linear and quadratic functions and show intercepts, maxima, and minima.

(*continued on the next page*)

Lesson Narrative

Previously, students used simple quadratic functions to describe how an object falls over time given the effect of gravity. In this lesson, they build on that understanding and construct quadratic functions to represent projectile motions. Along the way, they learn about the **zeros** of a function and the **vertex** of a graph. They also begin to consider appropriate domains for a function given the situation it represents.

Students use a linear model to describe the height of an object that is launched directly upward at a constant speed. Because of the influence of gravity, however, the object will not continue to travel at a constant rate (eventually it will stop going higher and will start falling), so the model will have to be adjusted. MP4

They notice that this phenomenon can be represented with a quadratic function, and that adding a squared term to the linear term seems to "bend" the graph and change its direction.

Instructional Routines

- Graph It
- Mathematical Language Routines
  - MLR8 Discussion Supports

Warm Up 6.1 Sky Bound (5 minutes)

In this warm up, students considers what happens if an object is launched up in the air unaffected by gravity. The work here serves two purposes. It reminds students that an object that travels at a constant speed can be described with a linear function. It also familiarizes students with a projectile context used in the next activity, in which students will investigate a quadratic function that more realistically models the movement of a projectile—with gravity in play.

Students who use a spreadsheet to complete the table practice choosing tools strategically. **MP5**

Standards Alignment

Addressing HSF-BF.A.1

Lesson 6-6

Building Quadratic Functions to Describe Situations (Part 2)

NAME ____________ DATE ____________ PERIOD ____________

Learning Goal Let's look at the objects being launched in the air.

Warm Up
6.1 Sky Bound

A cannon is 10 feet off the ground. It launches a cannonball straight up with a velocity of 406 feet per second.

Imagine that there is no gravity and that the cannonball continues to travel upward with the same velocity.

1. Complete the table with the heights of the cannonball at different times.

Seconds	0	1	2	3	4	5	t
Distance Above Ground (Feet)	10	416	822	1,228	1,634	2,040	$10 + 406t$

2. Write an equation to model the distance in feet, d, of the ball t seconds after it was fired from the cannon if there was no gravity. $d = 10 + 406t$

Student Edition, p. 245

Launch

Ask a student to read the opening paragraph of the activity aloud. To help students visualize the situation described, consider sketching a picture of a cannon pointing straight up, 10 feet above ground. Ask students to consider what a speed of 406 feet per second means in more concrete terms. How fast is that?

Students may be more familiar with miles per hour. Tell students that the speed of 406 feet per second is about 277 miles per hour.

Consider arranging students in groups of 2 so they can divide up the calculations needed to complete the table. Provide access to calculators, if requested.

Activity Synthesis

Ask students how the values in the table are changing and what equation would describe the height of the cannonball if there were no gravity. Even without graphing, students should notice that the height of the cannonball over time is a linear function given the repeated addition of 406 feet every time t increases by 1.

Tell students that, in the next activity, they will look at some actual heights of the cannonball.

Activity 6.2 Tracking a Cannonball (15 minutes)

Prior to this course, students learned that an object traveling at a constant speed can be described with a linear function whose graph is a straight line. Here they see a model that accounts for the fact that an object that is launched straight up at a constant speed does not keep going at the same rate when the influence of gravity is taken into account. Adding a quadratic term to a linear function has an effect of "bending" the graph, as the output values are no longer changing at a constant rate.

If students are unsure how to write an equation to represent the values in the table, ask them to compare how the actual heights of the cannonball at each second (when t is 1, 2, 3, etc.) differ from those in the no-gravity case (as shown in the table in the warm up). Finding the differences between the two outputs (16, 64, 144, . . .) at the same input values should help students think of the numbers and the functions they saw in the previous lesson.

To generalize the relationship between time and distance, students reason repeatedly with numerical values and look for regularity. **MP8**

If students opt to use spreadsheet or graphing technology, they practice choosing appropriate tools strategically. **MP5**

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Mathematical Language Routines
  - **MLR8** Discussion Supports

Standards Alignment

Addressing HSF-BF.A.1.a, HSF-IF.C.7.a

Activity
6.2 Tracking a Cannonball

Earlier, you completed a table that represents the height of a cannonball, in feet, as a function of time, in seconds, if there was no gravity.

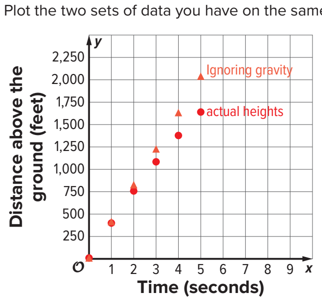
1. This table shows the actual heights of the ball at different times.

Seconds	0	1	2	3	4	5
Distance Above Ground (Feet)	10	400	758	1,084	1,378	1,640

Compare the values in this table with those in the table you completed earlier. Make at least 2 observations.

Sample observation:
- The actual height values are all smaller than the hypothetical ones (except the first, before the cannonball has been fired).
- The difference between the distances after some number of second is not constant. For example, 1 second after firing, the distances in the two tables are 416 and 400, a difference of 16 feet. Two seconds after firing, the difference between the two values is 64 feet.

2. Respond to each question.
   a. Plot the two sets of data you have on the same coordinate plane.



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Student Edition, p. 246

Launch

Arrange students in groups of 2. Give students a minute of quiet time to think about the first question, and then time to share their observations with their partner. Tell students that they will need to reference their work in the warm up.

Some students may choose to use a spreadsheet tool to extend the pattern, and subsequently to use graphing technology to plot the data. Make these tools accessible, in case requested.

Anticipated Misconceptions

When comparing the tables, some students may make observations that lack the detail needed to write an equation for the actual height. Prompt them rewrite the outputs for the actual height in terms of the hypothetical height ($400 = 416 - 16$, $758 = 822 - 64$, $1{,}084 = 1{,}228 - 144$, and so on). Show them values of $16t^2$ from a previous lesson to help them see and extend the pattern to write the equation.

(*continued on the next page*)

NAME ______________________ DATE ____________ PERIOD ____________

b. How are the two graphs alike? How are they different?

The dots (triangles) that form a straight line represent the hypothetical data that ignores gravity. The dots that form a curve represent the actual heights. They both start at the same place (0, 10). The hypothetical data show the heights increasing at a constant rate, in a linear pattern, while the actual data shows values growing more slowly each successive second.

3. Write an equation to model the actual distance d, in feet, of the ball t seconds after it was fired from the cannon. If you get stuck, consider the differences in distances and the effects of gravity from a previous lesson.

The effect of gravity is to subtract $16t^2$. An equation for the actual height of the cannonball is $d = 10 + 406t - 16t^2$.

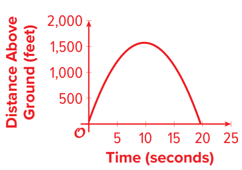
Activity
6.3 Graphing Another Cannonball

The function defined by $d = 50 + 312t - 16t^2$ gives the height in feet of a cannonball t seconds after the ball leaves the cannon.

1. What do the terms 50, $312t$, and $-16t^2$ tell us about the cannonball?

The 50 tells us the cannonball is fired from 50 feet off of the ground, the $312t$ tells us that the initial upward velocity is 312 feet per second, and $-16t^2$ represents the effect of gravity pulling the cannonball back to the ground.

2. Use graphing technology to graph the function. Adjust the graphing window to the following boundaries: $0 < x < 25$ and $0 < y < 2{,}000$.



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Activity Synthesis

Invite students to share their observations about the two tables (the one from the warm-up and the one here) and how the two graphs compare. Highlight responses that suggest that the values in the second table account for the effect of gravity.

Help students see how the output for each t value varies across the two tables. When t is 1, the output in feet in the second table is 16 less than in the first table. When t is 2, there is a difference of 64 feet. When t is 3, that difference is 144 feet, and so on. The values 16, 64, 144, . . . correspond to the expression $16t^2$ that we saw in the previous lesson (the distance fallen in feet as a function of time in seconds), so we can represent the values in the second table with the equation $d = 10 + 406t - 16t^2$. Ask students:

- **What do the 10, $406t$, and $-16t^2$ mean in this situation?** The 10 is the vertical position of the cannonball before it was launched: 10 feet above ground. In $406t$, the 406 tells us the vertical velocity at which it was shot up. The $-16t^2$ accounts for the effect of gravity on the height of the cannonball after it was shot up.
- **Why do you think the graph that represents $d = 10 + 406t$ changes from a straight line to a curve when $-16t^2$ is added to the equation?** Before that term was added the height increased by 406 feet every second. Adding $-16t^2$ decreases how much the cannonball travels up by some amount, but that amount gets larger each successive second. Eventually the cannonball stops increasing in height and starts to fall.

Support For English Language Learners

Conversing, Representing: MLR8 Discussion Supports. Use this routine to amplify mathematical uses of language to describe comparisons between the tables and graphs. After students share an observation, invite them to repeat their reasoning using mathematical language relevant to the lesson. Consider inviting the remaining students to repeat these phrases to provide additional opportunities for all students to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Support For Students with Disabilities

Representation: Internalize Comprehension. Use color-coding and annotations to highlight connections between representations in a problem. For example, ask students to highlight the actual height value and the hypothetical data values in the graph to show the contrast in values.

Supports accessibility for: Visual-spatial processing

Activity 6.3 Graphing Another Cannonball (15 minutes)

In this activity, students explore another model of a projectile motion. They graph and interpret a quadratic function in context and begin considering a reasonable domain for the function. Along the way, they practice reasoning concretely and abstractly. MP2

By the end of the lesson, they relate the **vertex** of the graph to the maximum height of the cannonball and the positive **zero** of the function to the time when the cannonball hits the ground.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Graph It

Standards Alignment

Addressing HSF-IF.B.5, HSF-IF.C.7.a

3. Observe the graph and:
   a. Describe the shape of the graph. What does it tell us about the movement of the cannonball?
   Sample response: The distance from the ground increases, reaches a peak, and then decreases which makes sense in the situation. It also looks like the distance from the ground decreases in the same way that it increased on the way up. The graph tells us the cannonball went up in the air, and then fell back down to the ground.
   b. Estimate the maximum height the ball reaches. When does this happen?
   Sample response: The cannonball gets up to a little over 1,500 feet, maybe to 1,600 feet.
   c. Estimate when the ball hits the ground.
   Sample response: The cannonball hits the ground a little bit before 20 seconds.
4. What domain is appropriate for this function? Explain your reasoning.
   The equation is a good model for the height of the cannonball from the time it is launched, $t = 0$, until the time it lands, just before $t = 20$. For times outside of these values, the function values do not mean anything in the context (for example, for $t = 21$ the function would place the cannonball underground).

Are you ready for more?

If the cannonball were fired at 800 feet per second, would it reach a mile in height? Explain your reasoning.

Sample response: The equation here would be $g(t) = 50 + 800t - 16t^2$. Since $g(10) = 6{,}450$ and there are 5,280 feet in a mile, this cannonball would reach over a mile in height. This could also be seen by graphing the function.

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Launch

Provide access to devices that can run graphing technology. If needed, demonstrate how to adjust the graphing boundaries of the graphing tool.

Depending on the graphing tool available and their facility with it, students may approach the estimations in the third question in different ways (including by eyeballing). If desired, demonstrate how to use the graphing tool to trace the graph and identify the coordinates of any point on it (which may include values that are precise or values rounded to a specified decimal place). Or, first observe how students go about estimating and give additional guidance as needed.

To support students with the last question, ask students: Is the equation a good model for predicting the height of the cannonball 10 seconds after it is fired? What about 1 minute after it is fired?

(*continued on the next page*)

Activity Synthesis

Invite students to share their observations and interpretations of the graph. Highlight the following points:

- The graph representing a quadratic function is a *parabola*, which is a special kind of U shape. We will learn more about the geometry of this shape in a later course, but for now, notice that there is a point when the graph changes direction, from going up as x increases to going down (or changes from going down to going up). We call this point the *vertex* of the graph.
- In the previous activity, we plotted a limited set of points, so we could not tell where the vertex of the graph was. In this graph, we are able to identify the vertex. In this situation, the vertex tells us the maximum height that the cannonball reaches and the number of seconds after launch that it took before it starts to fall.
- In this graph, we can also see that the height of the cannonball is 0 when t is a little less than 20. That point, the horizontal intercept, relates to the *zero* of the function, or an input value that produces 0 for the output. In this situation, the zero tells us when the cannonball hits the ground.
- Even though we can continue the graph beyond t of 20, in this situation any output values beyond that point would not have any meaning. After the cannonball hits the ground, the function is no longer appropriate for modeling the movement of the cannonball. Likewise, the function is not appropriate before $t = 0$ or before the cannon is fired. In this situation, a domain between 0 and just below 20 seconds is appropriate.

Support For Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: vertex of the graph, zero of the function, and parabola.

Supports accessibility for: Conceptual processing; Language

Lesson Synthesis (5 minutes)

To reinforce the connections between the parameters of a quadratic expression and the situation it describes, ask students:

- **So far, we've seen different expressions that represent vertical distances. Here are three expressions that all represent distance from the ground, in feet, as a function of time, in seconds. What does each of them tell us? Draw a diagram to illustrate the distances, if helpful.**
    - $16t^2$ the distance an object travels t seconds after being dropped
    - $400 - 16t^2$ the height of an object that is dropped from a height of 400 fee
    - $50 + 100t - 16t^2$ the height of an object that is shot up from 50 feet above the ground at a vertical speed of 100 feet per second, t seconds after being launched
- **If each expression defines a function, what does the zero of that function tell us?**
    - $16t^2$ The zero is the time when the object has traveled a distance of 0 feet. This happens at $t = 0$, before the object is dropped.
    - $400 - 16t^2$ The zero is the time when the height of the object is 0 feet, which is when it hits the ground.
    - $50 + 100t - 16t^2$ The zero is the time when the height of the object is 0 feet, which is also when it hits the ground.

Explain to students that the models seen here are simplified models and they ignore other factors such as air resistance, so the models that scientists use to study physical phenomena are likely to be more complex than what they've seen here.

If time permits, consider addressing a common misconception: that a graph of a quadratic function that represents distance-time relationship shows the physical trajectory of the object. Ask students to draw a sketch of what a bystander would see if they are facing the cannon as the ball is being launched.

Clarify that the graph represents the height of the object as a function of time, not the path that the object travels. In the examples given here, the object just goes straight up and straight down.

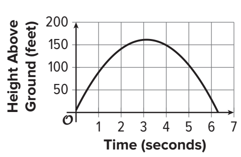
Cool Down 6.4 Rocket in the Air (5 minutes)

NAME ______ DATE ______ PERIOD ______

Lesson 6-6 Building Quadratic Functions to Describe Situations (Part 2)

Cool Down
6.4 Rocket in the Air

The height, h, of a stomp rocket (propelled by a short blast of air) above the ground after t seconds is given by the equation $h(t) = 5 + 100t - 16t^2$. Here is a graph that represents h.



1. How does the 5 in the equation relate to the graph?
   The graph intersects the vertical axis at 5.
2. What does $100t$ in the equation mean in terms of the rocket?
   It indicates that the initial velocity of the rocket was 100 feet per second upward.
3. What does the $-16t^2$ mean in terms of the rocket?
   This indicates the effect of gravity pulling the rocket back toward Earth.
4. About when does the rocket hit the ground?
   At about 6.3 seconds.

Printable Cool Down

Standards Alignment

Addressing HSF-IF.C

NAME ______________________ DATE ____________ PERIOD ____________

Summary
Building Quadratic Functions to Describe Situations (Part 2)

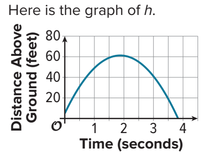
In this lesson, we looked at the height of objects that are launched upward and then come back down because of gravity.

An object is thrown upward from a height of 5 feet with a velocity of 60 feet per second. Its height $h(t)$ in feet after t seconds is modeled by the function $h(t) = 5 + 60t - 16t^2$.

- The linear expression $5 + 60t$ represents the height the object would have at time t if there were no gravity. The object would keep going up at the same speed at which it was thrown. The graph would be a line with a slope of 60 which relates to the constant speed of 60 feet per second.
- The expression $-16t^2$ represents the effect of gravity, which eventually causes the object to slow down, stop, and start falling back again.

Notice the graph intersects the vertical axis at 5, which means the object was thrown into the air from 5 feet off the ground. The graph indicates that the object reaches its peak height of about 60 feet after a little less than 2 seconds. That peak is the point on the graph where the function reaches a maximum value. At that point, the curve changes direction, and the output of the function changes from increasing to decreasing. We call that point the **vertex** of the graph.

Here is the graph of h.



The graph representing any quadratic function is a special kind of "U" shape called a *parabola*. You will learn more about the geometry of parabolas in a future course. Every parabola has a vertex, because there is a point where it changes direction—from increasing to decreasing, or the other way around.

The object hits the ground a little before 4 seconds. That time corresponds to the horizontal intercept of the graph. An input value that produces an output of 0 is called a **zero** of the function. A zero of the function h is approximately 3.8, because $h(3.8) \approx 0$.

In this situation, input values less than 0 seconds or more than about 3.8 seconds would not be meaningful, so an appropriate domain for this function would include all values of t between 0 and about 3.8.

Glossary

vertex (of a graph)
zero (of a function)

Practice
Building Quadratic Functions to Describe Situations (Part 2)

1. The height of a diver above the water is given by $h(t) = -5t^2 + 10t + 3$, where t is time measured in seconds and $h(t)$ is measured in meters. Select **all** statements that are true about the situation.

   A. The diver begins 5 meters above the water.

   B. The diver begins 3 meters above the water.

   C. The function has 1 zero that makes sense in this situation.

   D. The function has 2 zeros that make sense in this situation.

   E. The graph that represents h starts at the origin and curves upward.

   F. The diver begins at the same height as the water level.

2. The height of a baseball, in feet, is modeled by the function h given by the equation $h(t) = 3 + 60t - 16t^2$. The graph of the function is shown.

   a. About when does the baseball reach its maximum height?

   just before 2 seconds

   b. About how high is the maximum height of the baseball?

   about 60 feet

   c. About when does the ball hit the ground?

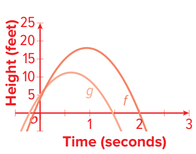
   less than 4 seconds after the baseball was hit or thrown

NAME ______________________ DATE ____________ PERIOD ____________

3. *Technology required.* Two rocks are launched straight up in the air. The height of Rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of Rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is time measured in seconds and height is measured in feet.

   Use graphing technology to graph both equations. Determine which rock hits the ground first and explain how you know.

   Rock B hits the ground first. Sample explanation: The graph for Rock B (function g) crosses the x-axis at about 1.5, which means it hits the ground about 1.5 seconds after launch. The graph for Rock A (function f) crosses the x-axis at about 2, which means it hits the ground about 2 seconds after launch.



4. Each expression represents an object's distance from the ground in meters as a function of time, t, in seconds.

   Object A: $-5t^2 + 25t + 50$

   Object B: $-5t^2 + 50t + 25$

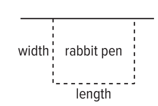
   a. Which object was launched with the greatest vertical speed?

   Object B

   b. Which object was launched from the greatest height?

   Object A

5. Tyler is building a pen for his rabbit on the side of the garage. He needs to fence in three sides and wants to use 24 ft of fencing. (Lesson 6-1)



   a. The table shows some possible lengths and widths. Complete each area.

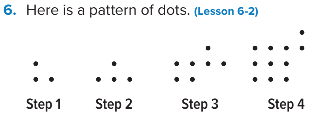
   See table.

Length (ft)	Width (ft)	Area (sq ft)
8	8	64
10	7	70
12	6	72
14	5	70
16	4	64

   b. Which length and width combination should Tyler choose to give his rabbit the most room?

   12 ft by 6 ft

6. Here is a pattern of dots. (Lesson 6-2)



Step	Total Number of Dots
0	3
1	4
2	7
3	12

   a. Complete the table.

   b. How many dots will there be in Step 10? 103

   c. How many dots will there be in Step n? $n^2 + 3$ or equivalent

Summary and Practice

NAME ______________________ DATE ____________ PERIOD ____________

7. The function f is defined by $f(x) = 2^x$ and the function g is defined by $g(x) = x^2 + 16$. (Lesson 6-4)

a. Find the values of f and g when x is 4, 5, and 6.

$f(4) = 16$, $f(5) = 32$, $f(6) = 64$, $g(4) = 32$, $g(5) = 41$, $g(6) = 52$.

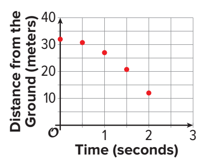
b. Will the values of f always be greater than the values of g? Explain how you know.

No. Sample response: The values of f are less than the values of g when x is 4 or 5, but when x is 6, the value of f is greater than the value of g. Because f is an exponential function that grows by a factor of 2, it will eventually overtake a quadratic function like g. For smaller values of x, the values of f will be smaller than the values of g.

8. Han accidentally drops his water bottle from the balcony of his apartment building. The equation $d = 32 - 5t^2$ gives the distance from the ground, d, in meters after t seconds. (Lesson 6-5)

a. Complete the table and plot the data on the coordinate plane.

t (Seconds)	d (Meters)
0	32
0.5	30.75
1	27
1.5	20.75
2	12



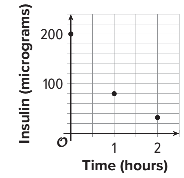
b. Is the water bottle falling at a constant speed? Explain how you know.

No. Sample explanation: The change in distance is not the same for equal intervals of time. Between 0 seconds and 0.5 seconds, the water bottle dropped 1.25 meters. Between 0.5 seconds and 1 second, the water bottle dropped 3.75 meters.

9. The graph shows how much insulin, in micrograms (mcg), is in a patient's body after receiving an injection. (Lesson 5-6)

a. Write an equation giving the number of mcg of insulin, m, in the patient's body h hours after receiving the injection.

$m = 200 \cdot \left(\frac{2}{5}\right)^h$



b. After 3 hours, will the patient still have at least 10 mcg of insulin in their body? Explain how you know.

Yes. After 3 hours, the patient has $200 \cdot \left(\frac{2}{5}\right)^3$ mcg of insulin in their body. This is almost 13 mcg.

Student Edition

Lesson 6-6

Building Quadratic Functions to Describe Situations (Part 2)

NAME ______________________ DATE ____________ PERIOD ____________

Learning Goal Let's look at the objects being launched in the air.

Warm Up

6.1 Sky Bound

A cannon is 10 feet off the ground. It launches a cannonball straight up with a velocity of 406 feet per second.

Imagine that there is no gravity and that the cannonball continues to travel upward with the same velocity.

1. Complete the table with the heights of the cannonball at different times.

Seconds	0	1	2	3	4	5	t
Distance Above Ground (Feet)	10						

2. Write an equation to model the distance in feet, d, of the ball t seconds after it was fired from the cannon if there was no gravity.

Activity

6.2 Tracking a Cannonball

Earlier, you completed a table that represents the height of a cannonball, in feet, as a function of time, in seconds, if there was no gravity.

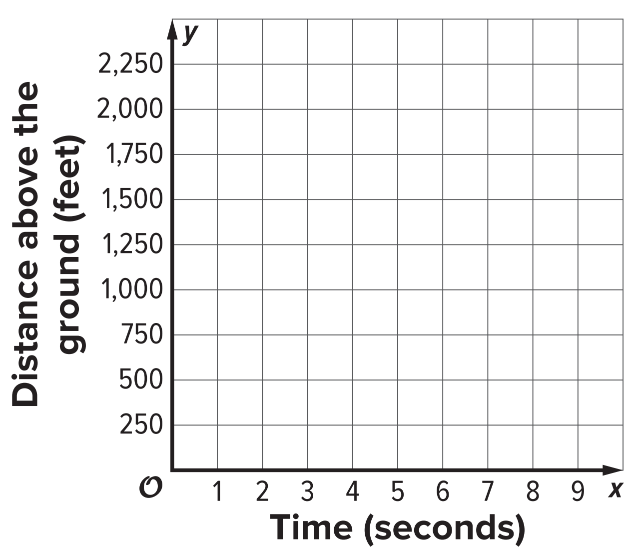
1. This table shows the actual heights of the ball at different times.

Seconds	0	1	2	3	4	5
Distance Above Ground (Feet)	10	400	758	1,084	1,378	1,640

Compare the values in this table with those in the table you completed earlier. Make at least 2 observations.

2. Respond to each question.

   a. Plot the two sets of data you have on the same coordinate plane.



NAME ______________________ DATE ____________ PERIOD __________

b. How are the two graphs alike? How are they different?

3. Write an equation to model the actual distance d, in feet, of the ball t seconds after it was fired from the cannon. If you get stuck, consider the differences in distances and the effects of gravity from a previous lesson.

Activity
6.3 Graphing Another Cannonball

The function defined by $d = 50 + 312t - 16t^2$ gives the height in feet of a cannonball t seconds after the ball leaves the cannon.

1. What do the terms 50, $312t$, and $-16t^2$ tell us about the cannonball?

2. Use graphing technology to graph the function. Adjust the graphing window to the following boundaries: $0 < x < 25$ and $0 < y < 2{,}000$.

3. Observe the graph and:

   a. Describe the shape of the graph. What does it tell us about the movement of the cannonball?

   b. Estimate the maximum height the ball reaches. When does this happen?

   c. Estimate when the ball hits the ground.

4. What domain is appropriate for this function? Explain your reasoning.

Are you ready for more?

If the cannonball were fired at 800 feet per second, would it reach a mile in height? Explain your reasoning.

NAME ______________________ DATE ____________ PERIOD ____________

Summary

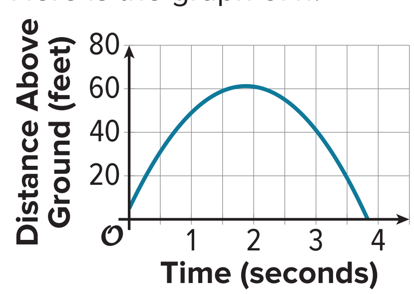
Building Quadratic Functions to Describe Situations (Part 2)

In this lesson, we looked at the height of objects that are launched upward and then come back down because of gravity.

An object is thrown upward from a height of 5 feet with a velocity of 60 feet per second. Its height $h(t)$ in feet after t seconds is modeled by the function $h(t) = 5 + 60t - 16t^2$.

- The linear expression $5 + 60t$ represents the height the object would have at time t if there were no gravity. The object would keep going up at the same speed at which it was thrown. The graph would be a line with a slope of 60 which relates to the constant speed of 60 feet per second.
- The expression $-16t^2$ represents the effect of gravity, which eventually causes the object to slow down, stop, and start falling back again.

Here is the graph of h.



Notice the graph intersects the vertical axis at 5, which means the object was thrown into the air from 5 feet off the ground. The graph indicates that the object reaches its peak height of about 60 feet after a little less than 2 seconds. That peak is the point on the graph where the function reaches a maximum value. At that point, the curve changes direction, and the output of the function changes from increasing to decreasing. We call that point the **vertex** of the graph.

The graph representing any quadratic function is a special kind of "U" shape called a *parabola*. You will learn more about the geometry of parabolas in a future course. Every parabola has a vertex, because there is a point where it changes direction—from increasing to decreasing, or the other way around.

The object hits the ground a little before 4 seconds. That time corresponds to the horizontal intercept of the graph. An input value that produces an output of 0 is called a **zero** of the function. A zero of the function h is approximately 3.8, because $h(3.8) \approx 0$.

In this situation, input values less than 0 seconds or more than about 3.8 seconds would not be meaningful, so an appropriate domain for this function would include all values of t between 0 and about 3.8.

Glossary

vertex (of a graph)
zero (of a function)

Practice

Building Quadratic Functions to Describe Situations (Part 2)

1. The height of a diver above the water is given by $h(t) = -5t^2 + 10t + 3$, where t is time measured in seconds and $h(t)$ is measured in meters. Select **all** statements that are true about the situation.

   A. The diver begins 5 meters above the water.

   B. The diver begins 3 meters above the water.

   C. The function has 1 zero that makes sense in this situation.

   D. The function has 2 zeros that make sense in this situation.

   E. The graph that represents h starts at the origin and curves upward.

   F. The diver begins at the same height as the water level.

2. The height of a baseball, in feet, is modeled by the function h given by the equation $h(t) = 3 + 60t - 16t^2$. The graph of the function is shown.

   a. About when does the baseball reach its maximum height?

   b. About how high is the maximum height of the baseball?

   c. About when does the ball hit the ground?

NAME ______________________ DATE ____________ PERIOD ____________

3. *Technology required.* Two rocks are launched straight up in the air. The height of Rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of Rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is time measured in seconds and height is measured in feet.

Use graphing technology to graph both equations. Determine which rock hits the ground first and explain how you know.

4. Each expression represents an object's distance from the ground in meters as a function of time, t, in seconds.

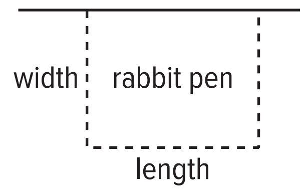
Object A: $-5t^2 + 25t + 50$

Object B: $-5t^2 + 50t + 25$

a. Which object was launched with the greatest vertical speed?

b. Which object was launched from the greatest height?

5. Tyler is building a pen for his rabbit on the side of the garage. He needs to fence in three sides and wants to use 24 ft of fencing. **(Lesson 6-1)**

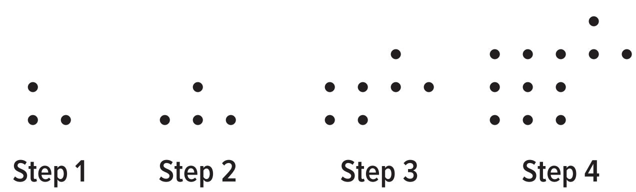


a. The table shows some possible lengths and widths. Complete each area.

Length (ft)	Width (ft)	Area (sq ft)
8	8	
10	7	
12	6	
14	5	
16	4	

b. Which length and width combination should Tyler choose to give his rabbit the most room?

6. Here is a pattern of dots. **(Lesson 6-2)**



Step	Total Number of Dots
0	
1	
2	
3	

a. Complete the table.

b. How many dots will there be in Step 10?

c. How many dots will there be in Step n?

NAME _______________ DATE _______________ PERIOD _______________

7. The function f is defined by $f(x) = 2^x$ and the function g is defined by $g(x) = x^2 + 16$. **(Lesson 6-4)**

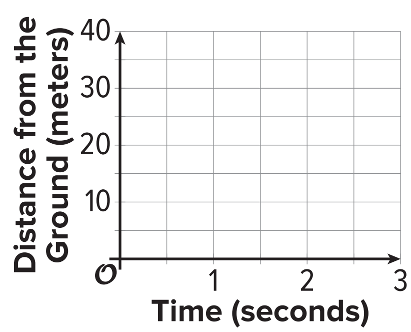
a. Find the values of f and g when x is 4, 5, and 6.

b. Will the values of f always be greater than the values of g? Explain how you know.

8. Han accidentally drops his water bottle from the balcony of his apartment building. The equation $d = 32 - 5t^2$ gives the distance from the ground, d, in meters after t seconds. **(Lesson 6-5)**

a. Complete the table and plot the data on the coordinate plane.

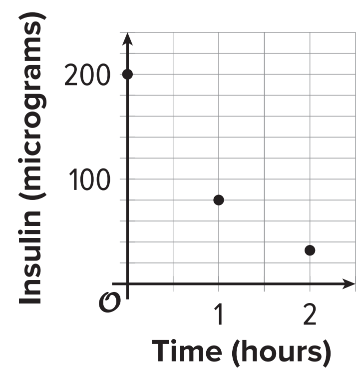
t (Seconds)	d (Meters)
0	
0.5	
1	
1.5	
2	



b. Is the water bottle falling at a constant speed? Explain how you know.

9. The graph shows how much insulin, in micrograms (mcg), is in a patient's body after receiving an injection. (Lesson 5-6)

a. Write an equation giving the number of mcg of insulin, m, in the patient's body h hours after receiving the injection.



b. After 3 hours, will the patient still have at least 10 mcg of insulin in their body? Explain how you know.

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