The Role of Discourse in Inclusive and Equitable Mathematics Learning and Teaching

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Introduction

Discourse is a fundamental aspect of learning and understanding mathematics at all grade levels and for all students. According to the National Council of Teachers of Mathematics (NCTM), "Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29).

This paper explores the what, why, and how of math discourse, emphasizing its crucial role in fostering deep understanding, enhancing argumentative skills, and promoting a culture of curiosity and active learning.

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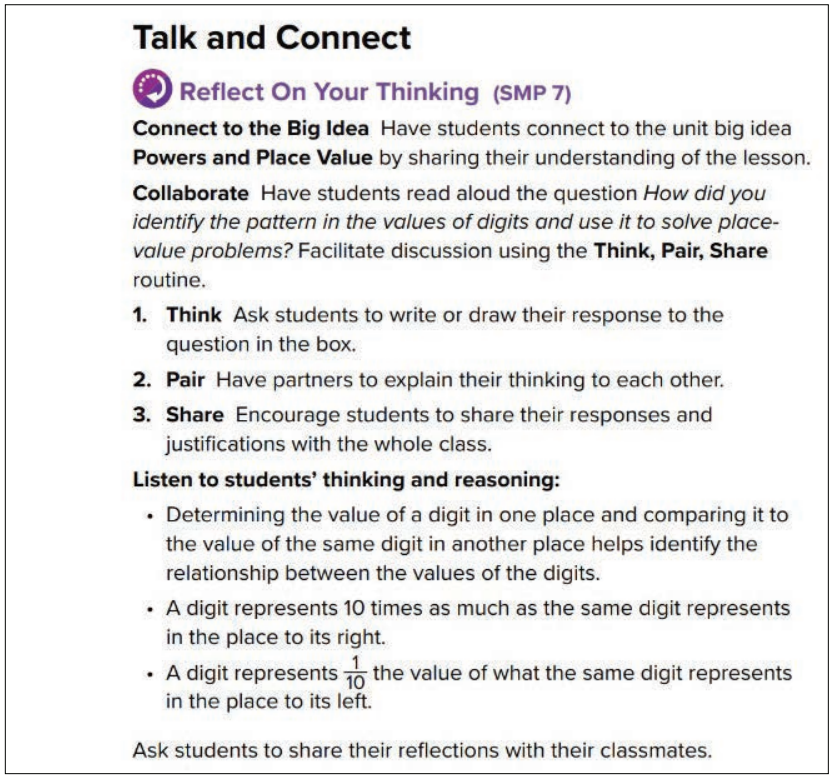
The Importance of Mathematical Discourse

When students participate in discourse, they articulate their thought processes, challenge their assumptions, and build on each other's ideas. This interactive approach not only deepens their understanding of mathematical concepts but also develops critical thinking and communication skills.

A primary benefit of math discourse is that it centers student thinking. Instead of passively receiving information from the teacher, students actively engage in the learning process by sharing their ideas, questioning assumptions, and exploring different perspectives. This student-centered approach promotes deeper understanding and fosters a sense of ownership and agency in learning. When students see their ideas valued and discussed, they are more likely to take risks, ask questions, and seek out new learning opportunities.

To center student thinking, teachers can use a variety of instructional strategies that encourage active participation and critical thinking. For example, the use of open-ended tasks and problems allows students to explore multiple solutions and to engage in meaningful mathematical discussions. Teachers can also use techniques such as think-pair-share and group problem-solving activities to facilitate student-centered discourse.



Talk and Connect

Reflect On Your Thinking (SMP 7)

Connect to the Big Idea Have students connect to the unit big idea **Powers and Place Value** by sharing their understanding of the lesson.

Collaborate Have students read aloud the question *How did you identify the pattern in the values of digits and use it to solve place-value problems?* Facilitate discussion using the **Think, Pair, Share** routine.

1. **Think** Ask students to write or draw their response to the question in the box.
2. **Pair** Have partners to explain their thinking to each other.
3. **Share** Encourage students to share their responses and justifications with the whole class.

Listen to students' thinking and reasoning:

- Determining the value of a digit in one place and comparing it to the value of the same digit in another place helps identify the relationship between the values of the digits.
- A digit represents 10 times as much as the same digit represents in the place to its right.
- A digit represents $\frac{1}{10}$ the value of what the same digit represents in the place to its left.

Ask students to share their reflections with their classmates.

Example from *California Reveal Math*® Grade 5 Teacher Edition

By creating a classroom environment where students are encouraged to share their thinking and to build on each other's ideas, teachers can help students develop a deeper and more comprehensive understanding of mathematical concepts.

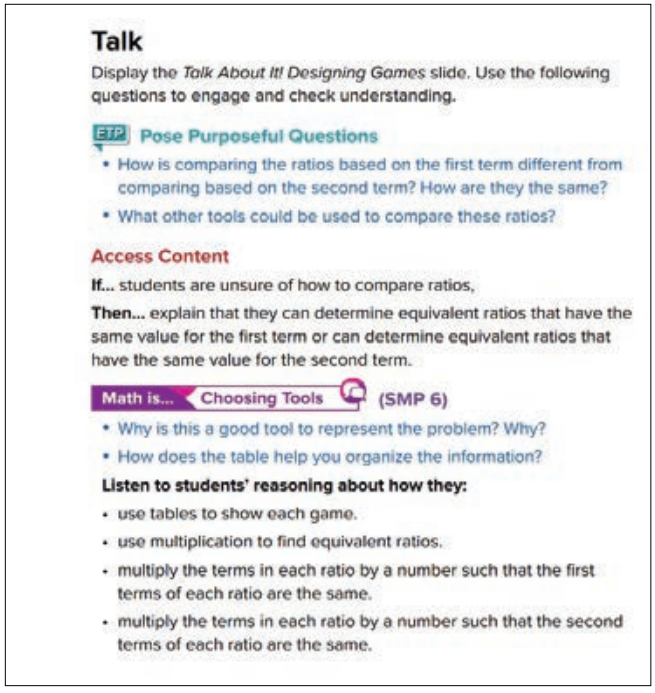
Connecting to the 2023 California Math Framework: SMPs

The Standards for Mathematical Practice (SMPs) embed the habits of mind and habits of interaction that form the basis of math learning—for example, reasoning, persevering in problem solving, and explaining one's thinking. To teach mathematics for understanding, it is essential to actively and intentionally cultivate students' use of the SMPs.

—Mathematics for All: Purpose, Understanding, and Connection, p. 23

The SMPs cultivate critical-thinking, problem-solving, and communication skills that are fundamental for understanding and applying mathematics effectively (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Mathematical discourse is a powerful tool that supports the development of skills in the SMPs. It encourages students to articulate their problem-solving approaches, discuss various strategies, and reflect on their thinking, thereby helping them persevere by providing support and alternative perspectives.

Discourse facilitates the exchange of ideas about the relationships between quantities and their abstract representations, allowing students to explain their reasoning and understand others' explanations. It also naturally involves students in presenting their arguments, justifying their reasoning, and engaging in critical discussions, which develops their ability to construct logical arguments and evaluate peers' reasoning. Discourse promotes discussions on how to represent real-world situations mathematically, helping students refine their ability to create and use mathematical models. Conversations about selecting and using mathematical tools effectively encourage students to share their experiences and learn to choose the most appropriate tools for specific tasks.



Talk

Display the *Talk About It! Designing Games* slide. Use the following questions to engage and check understanding.

ETP **Pose Purposeful Questions**

- How is comparing the ratios based on the first term different from comparing based on the second term? How are they the same?
- What other tools could be used to compare these ratios?

Access Content

If... students are unsure of how to compare ratios,

Then... explain that they can determine equivalent ratios that have the same value for the first term or can determine equivalent ratios that have the same value for the second term.

Math is... Choosing Tools **(SMP 6)**

- Why is this a good tool to represent the problem? Why?
- How does the table help you organize the information?

Listen to students' reasoning about how they:

- use tables to show each game.
- use multiplication to find equivalent ratios.
- multiply the terms in each ratio by a number such that the first terms of each ratio are the same.
- multiply the terms in each ratio by a number such that the second terms of each ratio are the same.

Example from *California Reveal Math*® Grade 6 Teacher Edition

Developing Norms for Idea-Focused Discussions

Creating a classroom culture that encourages open and respectful communication is essential for developing students' proficiency in constructing and critiquing arguments. Such an environment allows students to feel safe sharing their ideas and questioning others, leading to a deeper understanding of mathematical concepts.

Establishing a culture conducive to open, productive discourse involves:

- Setting clear expectations for discourse.
- Modeling respectful communication.
- Encouraging multiple strategies for solving problems.
- Providing continuous opportunities for students to engage in meaningful mathematical discussions.

Teachers play a pivotal role in this process by facilitating discussions, guiding students to articulate their reasoning, and helping them learn to listen and respond to their peers constructively.

Establishing norms in the classroom is crucial for fostering an environment conducive to effective mathematical discourse. Normalizing mistakes as learning opportunities rather than failures can foster students' willingness to engage in mathematical discourse. When students understand that errors are a natural part of the learning process, they are more likely to take risks, ask questions, and participate in discussions without fear of judgment. This approach not only enhances their argumentation skills but also cultivates a positive and inclusive learning environment. Respectful disagreement is another critical norm; teaching students to focus on ideas rather than individuals helps maintain a respectful and productive classroom atmosphere.

> Using phrases like "I see it differently because..." or "Can you explain why you think...?"
>
> encourages constructive dialogue and deepens understanding.

Encouraging multiple strategies for solving problems is also vital for promoting math discourse. Valuing and exploring different approaches highlights the richness of mathematical thinking and helps students appreciate diverse perspectives. This norm fosters creativity and critical thinking, allowing students to see that there is often more than one way to reach a solution.

Promoting equity of voice ensures that all students have the opportunity to contribute to discussions. This can be achieved by using strategies to give quieter students a chance to speak or by managing dominant voices to prevent them from overpowering the conversation. By establishing these norms, educators can create a dynamic and supportive environment where every student feels empowered to engage and share their mathematical thinking.

Developing and maintaining these norms involves consistently modeling and reinforcing the desired behaviors during classroom interactions. Teachers can demonstrate how to ask clarifying questions, provide constructive feedback, and build on others' ideas.

Connecting to the 2023 California Math Framework: Classroom Norms

Establishing classroom norms and routines can support students in attending to and making sense of their peers' mathematical ideas and questions in ways that position one another's thinking as worthy of taking into consideration (see also Cabana, Shreve, and Woodbury, 2014). Teachers must create norms and structures that enable all students to share and discuss ideas inclusively and draw students into mathematical conversations on an equal footing.

—Exploring, Discovering, and Reasoning With and About Mathematics, p. 41

Suggested Math Class Norms

1. Everyone can learn math to the highest levels.
2. Mistakes are valuable for learning.
3. Questions are important.
4. Math is about creativity and making sense.
5. Math is about connections and communicating.
6. Depth is more important than speed.
7. Math class is about learning with understanding.
8. Everyone has the right to share their thinking.
9. We learn more when we attend to and make sense of the thinking of others.
10. All cultures reflect histories of important mathematical thinking and applications.

—Exploring, Discovering, and Reasoning With and About Mathematics, p. 17

Routines and Strategies to Facilitate Mathematical Discourse

The National Council of Teachers of Mathematics' *Principles to Actions* emphasizes that effective teaching of mathematics involves facilitating discourse among students to build a shared understanding (NCTM, 2014). By analyzing and comparing different student approaches and arguments, students gain a deeper appreciation and comprehension of mathematical concepts. This shared understanding is cultivated through guided discussions where teachers help students articulate their reasoning, listen to others' perspectives, and synthesize various ideas into a coherent whole.

Providing opportunities for students to talk early and often about their math ideas is crucial for developing understanding and ownership of their learning. By encouraging students to share their thoughts from the beginning of the lesson, teachers can create an environment where every voice is valued and mathematical reasoning is continuously explored. Early discussions can be viewed as "rough draft thinking" as students share and discuss their initial, unpolished ideas which are later refined through peer feedback and collaborative revision (Jansen, 2023). Frequent opportunities for discourse throughout the lesson help students refine their ideas, clarify misunderstandings, and deepen their conceptual understanding.

To cultivate an engaging and effective mathematical discourse environment, educators can implement key practices for utilizing student responses with structured techniques like math talk moves and supportive peer discussion routines.

Promoting meaningful whole-class discussions

Smith and Stein (2011) outlined five key practices for effectively using student responses in whole-class discussions:

1. Anticipate potential student responses before the lesson begins.
2. Actively monitor students' work and engagement during the lesson.
3. Select specific students to present their mathematical work.
4. Strategically sequence the responses for the discussion.
5. Make connections between different students' responses and link these responses to key mathematical concepts.

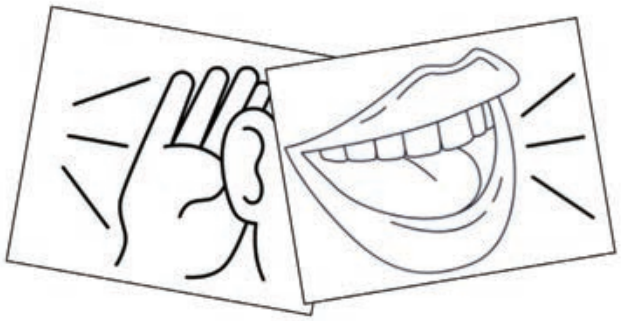
These practices support teachers in fostering rich mathematical discourse by encouraging deeper student engagement and understanding. By anticipating responses, teachers can better prepare for diverse perspectives, while monitoring allows them to gauge comprehension in real time. Selecting and sequencing responses not only values student contributions but also creates a cohesive narrative in discussions, ultimately helping students make connections to key mathematical ideas. This structured approach empowers educators to facilitate productive conversations that enhance learning outcomes and promote a collaborative classroom culture.

Facilitating student discourse

The use of structured techniques such as "math talk moves" (Chapin et al, 2022) supports the process of sharing and refining ideas. Math talk moves refer to specific strategies or techniques to facilitate meaningful mathematical discourse used by both the teacher and students. Here are some common math talk moves:

- **Revoicing questions** can begin with, "So, you're saying that...?" or "Let me see if I understand. You think that...?" This move confirms the student's idea and can prompt others to engage with it more deeply.

- **Repeating questions** often begin with, "Can you say that again?" or "What did [another student] say?" This move helps ensure that all students have heard and understood the key ideas being discussed.
- **Probing questions** often begin with, “Can you explain why…?" or "What makes you think that…?" This approach helps students articulate their thought processes and examine their ideas more critically, fostering deeper engagement with the mathematical concepts being discussed.
- **Prompting questions** often begin with phrases like, "What do you notice about…?" or "How could you figure out…?" This technique helps students focus their thinking and explore mathematical ideas more effectively.
- **Connecting questions** often begin with prompts like, "How is this similar to…?" or "Can you think of another way to represent…?" These questions help students make connections between different mathematical ideas, representations, or strategies.



Talk & Listen Cards
(Keeley & Tobey, 2017)

Peer interactions

Peer discussions provide students with a platform to share their ideas, challenge each other's thinking, and build on their collective knowledge. Even the youngest learners can engage in peer discussions, benefiting from the opportunity to articulate their thoughts and hear diverse perspectives. The use of structured routines supports peer interactions. For example, Turn and Talks using Talk & Listen Cards (Keeley & Tobey, 2017) can support Kindergarten conversations, as physically holding and swapping the cards provides a tangible reminder of the roles and expectations during the discussion. These routines ensure that each student has a chance to speak and listen, fostering equitable participation and enhancing the quality of their mathematical discourse.

Connecting to the 2023 California Math Framework: Peer Revoicing

Peer revoicing is a powerful routine for promoting shared understanding of mathematics as well as mutual recognition as young mathematicians. It structures the dialogue between the speaker and the listener in a way that ensures that the contributions build meaningfully upon each other. Teacher and peer revoicing can elevate the mathematical contributions of a student perceived as low-status (Cohen and Lotan, 1997; Cabana, Shreve, and Woodbury, 2014; LaMar, Leshin, and Boaler, 2020).

—Teaching for Equity and Engagement, p. 36

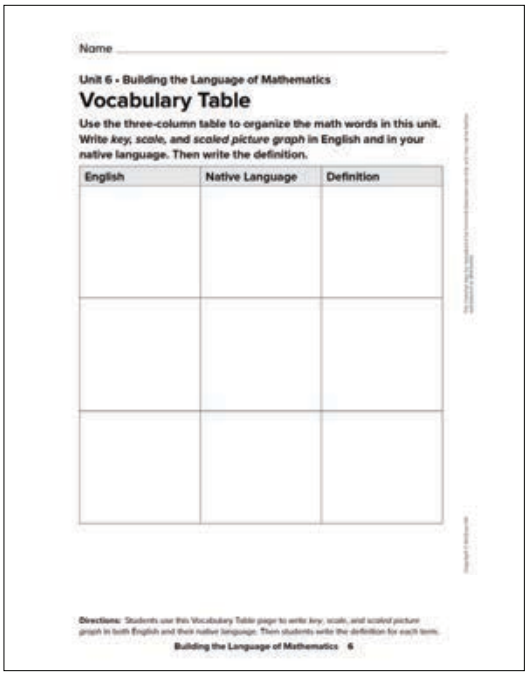
Discourse and Intentional Language Development

High-quality discourse requires intentional planning, especially in language development. Opportunities for language development help students use precise mathematical language, enabling them to articulate their thinking clearly and accurately. This focus is particularly important for students learning English as an additional language (Callahan et al., 2020). Language development encompasses more than just words, phrases, vocabulary, and definitions; it includes teaching students to use a full range of discourse tools, including multiple mathematical representations (Håkansson, 2017). By emphasizing precise terminology and multiple representations, and by providing numerous communication opportunities, teachers can help all students develop a deeper and more accurate understanding of mathematical concepts.

To effectively support students' mathematical language development, educators can implement a range of strategies that integrate language objectives, foster collaborative learning, and utilize structured routines.

Strategies

Strategies for supporting mathematical language development include using visual aids, encouraging the use of native language, fostering collaborative learning, providing sentence frames, and explicitly teaching mathematical vocabulary. The use of sentence stems and frames provide students with scaffolding to help them structure their responses and use appropriate mathematical language. For example, sentence stems like "I agree with ____ because ____" or "One possible solution is ____ because ____" can guide students in articulating their reasoning and engaging in productive discourse. Providing visual supports such as word walls and graphic organizers can help students utilize key vocabulary and concepts. By incorporating these strategies, educators can enhance comprehension and participation in math discourse, ultimately improving mathematical understanding and language proficiency (Banse, et al, 2016).



Name _____

Unit 6 • Building the Language of Mathematics

Vocabulary Table

Use the three-column table to organize the math words in this unit. Write *key*, *scale*, and *scaled picture graph* in English and in your native language. Then write the definition.

English	Native Language	Definition

Directions: Students use this Vocabulary Table page to write *key*, *scale*, and *scaled picture graph* in both English and their native language. Then students write the definition for each term.

Building the Language of Mathematics 6

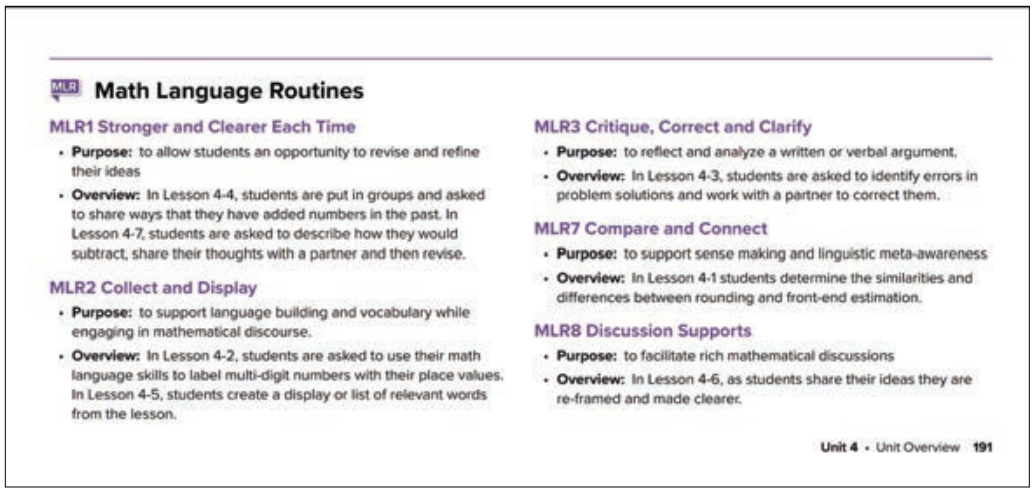
Example from *California Reveal Math*® Grade 3.

Language Objectives

Incorporating language objectives helps ensure that language development is integrated with content learning. By setting clear language goals and explicitly teaching the language structures and vocabulary needed for mathematical discourse, teachers can help students become more proficient communicators. This intentional focus on language supports English learners and students from diverse linguistic backgrounds and benefits all students by promoting clarity and precision in mathematical discussions.

Structured Routines

Mathematical language routines (MLRs) are structured formats that help students develop their mathematical language, content, and practices. MLRs support discourse by aiding students in making sense of math and enhancing their understanding of language, skills, and mathematical concepts. By amplifying language, students can acquire more sophisticated terms and concepts. MLRs also improve expression by helping students articulate their thoughts more clearly when writing and speaking about math. Additionally, these routines cultivate conversation, encouraging rich interactions that fill knowledge gaps, expose mistakes, and facilitate collective correction (Zwiers et al, 2017).



Math Language Routines

MLR1 Stronger and Clearer Each Time

- **Purpose:** to allow students an opportunity to revise and refine their ideas
- **Overview:** In Lesson 4-4, students are put in groups and asked to share ways that they have added numbers in the past. In Lesson 4-7, students are asked to describe how they would subtract, share their thoughts with a partner and then revise.

MLR2 Collect and Display

- **Purpose:** to support language building and vocabulary while engaging in mathematical discourse.
- **Overview:** In Lesson 4-2, students are asked to use their math language skills to label multi-digit numbers with their place values. In Lesson 4-5, students create a display or list of relevant words from the lesson.

MLR3 Critique, Correct and Clarify

- **Purpose:** to reflect and analyze a written or verbal argument.
- **Overview:** In Lesson 4-3, students are asked to identify errors in problem solutions and work with a partner to correct them.

MLR7 Compare and Connect

- **Purpose:** to support sense making and linguistic meta-awareness
- **Overview:** In Lesson 4-1 students determine the similarities and differences between rounding and front-end estimation.

MLR8 Discussion Supports

- **Purpose:** to facilitate rich mathematical discussions
- **Overview:** In Lesson 4-6, as students share their ideas they are re-framed and made clearer.

Unit 4 • Unit Overview 191

Example from *California Reveal Math*® Grade 4 Teacher Edition

Connecting to the 2023 California Math Framework: Intentional Language Development

The facilitation of high-quality discourse needs to be intentional, especially with regard to language development.

—Exploring, Discovering, and Reasoning With and About Mathematics, pp. 19–20

The Math Language Routines, developed by Understanding Language at the Stanford Center for Assessment, Learning, and Equity, provide teachers with a set of robust routines to foster student participation while simultaneously building math language, practices, and content.

—Exploring, Discovering, and Reasoning With and About Mathematics, p. 49

In a “Critique, Correct, Clarify” activity, students are provided with teacher-made or curated ambiguous or incomplete mathematical arguments (e.g., “1/2 is the same as 3/6 because you do the same to the top and bottom” or “2 hundreds is more than 25 tens because hundreds are bigger than tens”). Students practice respectfully making sense of, critiquing, and suggesting revisions together.

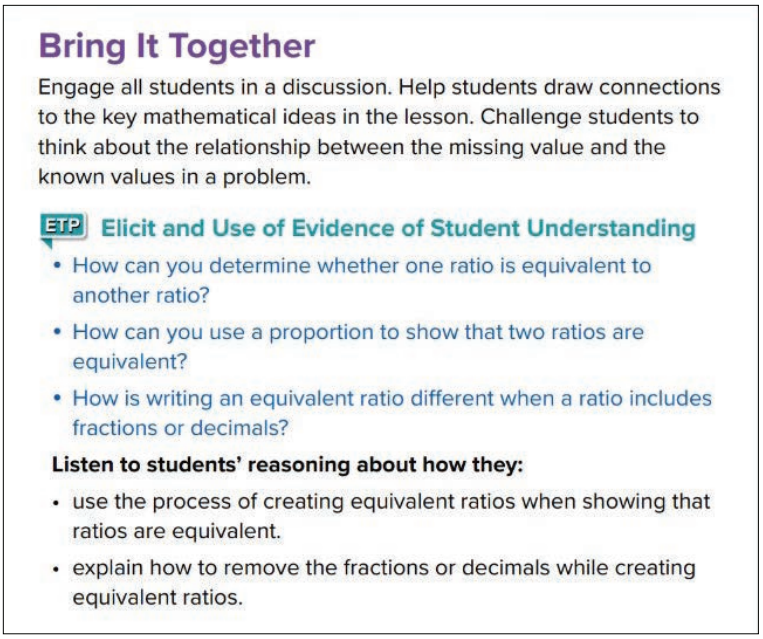
—Exploring, Discovering, and Reasoning With and About Mathematics, pp. 37–38

Discourse and Formative Assessment

Formative assessment is the process of using evidence to inform instruction and support learning, with instructional decisions made by the teacher and learning decisions made by the student at the heart of the process (Keeley & Tobey, 2017). By integrating thoughtful and structured student interactions into their teaching, educators can gain insights into students’ mathematical understanding, problem-solving strategies, and misconceptions (Cirillo & Langer-Osuna, 2018). A treasure trove of formative assessment data can be mined merely by carefully listening to students talk about their ideas and justify their thinking. Evidence gleaned from these interactions allows educators to gather real-time data on student learning, which is pivotal for making informed instructional decisions and providing targeted feedback to address specific learning gaps.

Effective use of classroom discussions in formative assessment involves asking open-ended questions, promoting peer interaction, and actively monitoring and documenting student conversations to gather valuable data for assessment and reflection. Ensuring that these interactions are aligned with the lesson's learning and language goals helps teachers gauge whether students are meeting these goals and allows for instructional adjustments as needed. Additionally, feedback based on observations made during discussions should be specific, timely, and focused on helping students improve their understanding and skills, guiding them toward further learning.

These strategies collectively foster an environment where formative assessment through student interactions can significantly enhance learning outcomes in mathematics. Attending to the connection between eliciting and probing student thinking through discourse moves, making sense of and responding to student thinking revealed during discussions, and attending to the dynamics of participation to assess equitable opportunities supports discourse that is purposeful, productive, and powerful (Cirillo & Langer-Osuna, 2018).



Bring It Together

Engage all students in a discussion. Help students draw connections to the key mathematical ideas in the lesson. Challenge students to think about the relationship between the missing value and the known values in a problem.

ETP Elicit and Use of Evidence of Student Understanding

- How can you determine whether one ratio is equivalent to another ratio?
- How can you use a proportion to show that two ratios are equivalent?
- How is writing an equivalent ratio different when a ratio includes fractions or decimals?

Listen to students' reasoning about how they:

- use the process of creating equivalent ratios when showing that ratios are equivalent.
- explain how to remove the fractions or decimals while creating equivalent ratios.

Example from *California Reveal Math*® Grade 7 Teacher Edition

Connecting to the 2023 California Math Framework: Formative Assessment

One of the strengths of formative assessment is the flexibility that it affords a classroom teacher, both in timing and approach. Indeed, one can argue that there are myriad possibilities for teachers to conduct formative assessment throughout a lesson, such as monitoring the types of questions students ask, the responses students share to questions, and the quality of content in peer conversations. And—though much of this may be unplanned—when formative assessment is intentionally included in a daily lesson plan, the data and analysis are even more effective.

—Mathematics Assessment in the 21st Century, pp. 17–18

Conclusion

Mathematical discourse is a powerful tool for enhancing student learning and engagement in mathematics. By fostering a classroom culture that values communication, collaboration, and critical thinking, teachers can help students develop a deeper and more comprehensive understanding of mathematical concepts. Through intentional planning and the use of effective instructional strategies, teachers can promote meaningful discourse that centers student thinking, supports language development, and encourages the construction of mathematical arguments. By prioritizing mathematical discourse, educators can create a dynamic and inclusive learning environment that promotes curiosity, active learning, and a lifelong love of mathematics.

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