Core-Plus Mathematics

Design and Development

The Core-Plus Mathematics Project has completed a revised edition of its problem-based, inquiry-oriented, technology-rich four-year curriculum. Revisions were informed by recent research on student learning, continuing feedback from teachers using the curriculum materials, and the Common Core State Standards for Mathematics (CCSS), with which the new edition is strongly aligned. The CCSS Edition of *Core-Plus Mathematics* builds on the strengths of the first edition that was recognized by the U.S. Department of Education as one of six exemplary mathematics programs in the U.S., and the updated and refined second edition that was recognized by the American Institute for Research and the Business-Higher Education Forum's Strategic Ed Solutions as one of 35 education programs in the U.S. (across all subject areas) that increase student achievement and improve college and career readiness.

Genesis

The Core-Plus Mathematics Project was initially funded in 1992 by the National Science Foundation to develop a comprehensive *Standards*-based mathematics curriculum for grades 9–12. From the outset, our goal was to create a high school curriculum that would enable schools to successfully negotiate many of the difficult challenges of curricular reform outlined in *Everybody Counts* (NRC 1989), particularly:

- expanding the traditional vision of school mathematics that provided minimal mathematics for the majority and advanced mathematics for a few to provide a significant core of important mathematics to all students;
- retaining the goal of strong preparation for future studies in mathematics and its applications, but increasing attention to mathematical topics that are relevant to students' present and future needs.

The framework for the intended curriculum consisted of a single three-year core sequence of broadly useful mathematics for both college-bound and employment-bound students, plus a flexible fourth-year course that continued the preparation of students for college. Course 4 would consist of a core of four units for all college-bound students, plus

---

This paper is an expansion and update of a chapter, "The Case of Core-Plus Mathematics" (Fey and Hirsch, 2007), in *Perspectives on the Design and Development of School Matheatmics Curricula* (Hirsch, ed., 2007). It is based on work supported, in part, by the National Science Foundation under grants no. MDR-9255257, ESI-9618193, and ESI-0137718. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Foundation.

additional units enabling tailoring of classes to support further mathematical preparation for either calculus-based or non-calculus-based undergraduate programs.

The team of mathematics educators working on the *Core-Plus Mathematics* program viewed the curriculum development process as an extended design experiment (Brown 1992; Collins 1992; Design-Based Research Collective 2003; Gravemeijer 1994) that includes iterative cycles of curriculum material design, development, field testing, evaluation, and revision. Additionally, we viewed professional development support for teachers and their effective implementation of the curriculum to be an important part of our development responsibility.

Creation of any comprehensive school mathematics curriculum requires decisions that reflect the developers' understanding of mathematics as a discipline, their knowledge of research on learning and teaching, and their experience in schools. Advice and deliberation with those considerations led to design principles for the content and shape of the intended curriculum. Development of detailed materials reflecting that vision required imaginative and careful creation of plans and materials reflecting design principles.

Design Principles

Creation of the Core-Plus Mathematics curriculum and instructional materials has been guided by the development team's shared understanding of mathematics and its teaching and learning. There are at least five key dimensions of that vision.

Mathematical Content

Development of the Core-Plus Mathematics curriculum began with our belief that the essence of mathematics is its concepts and reasoning methods for making sense of our observations and experiences in the real world. Concepts like similarity, equation, distribution, and network help us to describe and analyze visual, numeric, and stochastic patterns. Operators like transformations, functions, logical inference rules, and matrices help us to extend patterns and solve problems. This view of mathematics as a vibrant and broadly useful subject that can be best learned and understood as an "active science of patterns" (Steen 1990) led us to several important design principles that shaped the scope and sequence of mathematical topics in the Core-Plus Mathematics curriculum:

- The curriculum should include major content strands developing concepts and skills in algebra/functions, statistics/probability, geometry/trigonometry, and discrete mathematics;
- In addition to learning specific facts, principles, and procedures associated with the core content strands, the curriculum should pay explicit attention to developing students' mathematical habits of mind (Cuoco et al. 1996) like visualizing, searching for and explaining patterns, thinking recursively, modeling, optimizing, and justifying and proving.

- The curriculum should explicitly develop student understanding and skill in use of mathematical modeling, including the processes of data collection, representation, interpretation, prediction, and simulation.

To the extent feasible within educational policy constraints of common school conditions, we have worked hard to follow these guidelines for development of the mathematical content in the four-year curriculum.

Curriculum Sequence and Organization

Choice of mathematical concepts and skills to be developed was an important first step in mapping our curriculum framework. But the grade placement and ordering of mathematical topics was not a trivial task. Our work on this problem has been guided by the following key principles:

- The curriculum should include significant and broadly useful topics from each content strand in each year of the program.
- Topics from the separate content strands should be developed in coherent, focused units that exploit useful connections to the other strands.
- The mathematical content in any year of the curriculum should reflect judgments of what would be most important for students to know if that was to be their last formal experience in school mathematics.

Commitment to these curriculum sequence and organization principles has led us to a program that does indeed treat each main content strand in each year. Furthermore, we have departed from long-standing curricular traditions by placing topics where they seem most natural and important, rather than where they have "always been." For instance, our development of formal algebraic symbol manipulation was gradually built up over all four years of the curriculum, rather than being concentrated in the first and third high school years. More complex manipulative skills were delayed to a point later in the curriculum when they would be needed by some students in their future work. Our developments of formal logical inference and geometric proof occur somewhat later than tradition would dictate, although informal reasoning and justification of results occur earlier and in more pervasive ways than in conventional U.S. curricula. Similarly, topics such as recursion, matrices, simulation (Monte Carlo) methods, and geometric transformations (including coordinate representations) occur earlier and in more pervasive ways than in more traditional curricula.

Role of Technology

At the outset of our work on the *Core-Plus Mathematics* program we were cautioned by teachers about making strong assumptions of access to technology like computers. Thus, we designed the first edition of CPMP curriculum materials with an assumption that students would only have ready access to graphing calculators. The fundamental aim in our use of technology was to enable multiple representations of mathematical ideas and to support a variety of robust strategies for mathematical thinking and quantitative problem solving.

Our first edition curriculum design and development did not assume ready access to emerging dynamic geometry tools or computer algebra systems. However, in the revision of the Core-Plus Mathematics program that is described later, we are making a more significant commitment to use of interactive geometry software, spreadsheets, computer algebra systems, data analysis and simulation software, and tools for exploratory work in graph theory.

To some extent, our judgments about the reasonable impact of calculator and computer tools have been influenced by our choices of intended implementation timeframe. If our aim had been to develop curriculum ideas and materials that might be accepted for common school use by 2010 or later, we might have been more adventurous. However, our conscious goal in development of the first edition of *Core-Plus Mathematics* was to produce curriculum materials that would help schools implement recommendations of the 1989 NCTM *Curriculum and Evaluation Standards* and later the up-date described in the 2000 *Principles and Standards for School Mathematics*.

Instructional Design

Traditional conceptions of mathematics education assign textbooks responsibility for clear and concise explanations of mathematical ideas and procedures, with illustrative examples to guide student work on subsequent practice exercises. Of course, this model for curriculum materials is based on a conception of instruction that assigns the same role to teachers. Imaginative teachers have always enhanced textbooks presentations by designing lessons that engaged students more actively in exploration and discovery of mathematical principles—believing that students learn and remember best those ideas that they have sorted out for themselves.

Design and development of the Core-Plus Mathematics curriculum materials were based on an explicit intention to support problem-based, student-centered classroom activity informed by theory and research on teaching and learning. As we developed materials for individual units and lessons, we aimed to implement the following instructional principles:

- Introduction of new mathematical ideas will be most effective if they appear in problem contexts that students can relate to and that connect to their prior knowledge. Authentic applied problems are especially useful contexts for learning, but significant pure mathematical problems are often useful also. (Hiebert et al. 1996)
- Effective mathematics instruction engages students in collaborative small-group investigations of problem situations that encourage student-to-student dialogue, followed by teacher-led whole group summarizing activities that lead to analysis, abstraction, and further application of underlying mathematical ideas (Cobb 1995; Davidson and Kroll 1991). There is also some evidence that small-group

collaborative learning encourages a range of social skills conducive to the learning styles of groups that are currently underrepresented in mathematics. (Oakes 1990)

- Students should be regularly involved in mathematical activities like searching for patterns, making and verifying conjectures, generalizing, applying, proving, and reflecting on the process. (Freudenthal 1983)

Commitment to these instructional design principles led us to production of curriculum materials that intend support of a modified *launch–explore–summarize* instructional model. For each lesson, an introductory problem situation sets the context for the class investigation and it is used for initial discussion involving the teacher and the whole class. Then students work in small groups to solve short sequences of selected problems and summarize their findings in response to several post-investigation questions that are discussed again with the teacher and the whole class. Homework tasks following lessons are designed to engage students in applying, organizing, reflecting on, and extending their evolving mathematical understanding.

The instructional model for which Core-Plus Mathematics curriculum material is written has led us to student text material that is a thoughtfully constructed sequence of problems, not an archive of results and illustrative worked examples. The investigations often involve hands-on experiments, data analysis and modeling, and technology-based exploration of patterns.

For example, an introductory lesson on geometry of three-dimensional shapes begins with an experiment in which students test the load-bearing capacity of columns with constant perimeter but varying shapes. After collecting and organizing experimental data, students look for a numerical pattern relating number of sides in the polygonal base shape to column capacity. The lesson makes the key point that geometric form and function are closely related and it also connects geometric properties to algebraic representations and numeric patterns.

Assessment Principles

Assessing what students know and are able to do is an integral part of the Core-Plus Mathematics curriculum's instructional model. There are opportunities for formative assessment in each phase of the instructional cycle. First, Think About This Situation questions in the lesson launch activities allow teachers to assess the prior knowledge that students bring to investigation of a new topic. While students work in small groups to explore new ideas and solve problems, teachers are able to monitor the group work and see where students are gaining insights or having problems. An end-of-investigation Checkpoint that provides explicit questions designed to guide whole-class summary of investigation results offers another opportunity to check student progress. Those questions are followed by short On Your Own problems that give quick feedback on student grasp of the new material.

Since the student text is not a reference book of complete results (there are no answers to investigation or homework problems given in the student text), students are prompted in several ways to construct their own summaries of key ideas and to record them in a mathematical toolkit and journal that evolves over time. This record of student thinking and understanding provides them and their teachers with another useful tool to monitor progress of learning.

In addition to support for continual monitoring of student progress during instruction, we also provide materials for teachers to use or adapt as quizzes, unit examinations, project and take-home assessments, and mid-term and final summative assessments.

Development Process

The overall scheme of curriculum development in the Core-Plus Mathematics Project and other *Standards*-based reform projects of the past decade is similar in many respects to the iterative process outlined in the literature of design research (Design-Based Research Collective 2003), design experiments (Brown 1992; Collins 1992), developmental research (Gravemeijer 1994), and engineering research (Burkhardt and Schoenfeld 2003).

The Core-Plus Mathematics curriculum was developed in consultation with an international advisory board, mathematicians, instructional specialists with expertise in equity and access issues, and classroom teachers. Each course was the product of a four-year research, development, and evaluation process. After a year of initial development with perhaps some local trials, the pilot version of a course was tested in 19 Michigan high schools. During this pilot year, CPMP teachers in these schools provided extensive feedback to the authors by noting what worked and what was in need of revision. In addition, students in the CPMP classes were pretested at the beginning of Course 1 and posttested at the end of Course 1 and also at the end of each course thereafter. The needed revisions identified by pilot teacher comments and test results were made promptly so that a revised, field-test version of a course was ready for use during the following school year.

The third year of development was the national field test, conducted in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. A broad cross-section of students from urban, suburban, and rural communities with ethnic and cultural diversity was represented. Evaluative data, including evaluator field notes, teacher-annotated units, and focus group meetings with field-test teachers were used by the authors to make further revisions in the materials before they were finally published for wide-scale use.

The orderly cycle of design, development, testing, evaluation, and revision described in theory is seldom matched in practice. In fact, our experience suggests that, while the overall scheme follows the theory, there is almost continual interaction among all aspects of the process as a wide variety of opinions, advice, and external conditions are imposed on the development process.

The most carefully considered and widely agreed upon outline of a mathematical development for a unit or course often looks much less attractive when authors try to create problem material that will support the planned sequence of topics. Field-test focus groups often provide feedback on one unit that dictates changes with serious implications for other units. Emerging state standards introduce new considerations into the discussion of grade placement and topic coverage.

In the experience of developing the Core-Plus Mathematics curriculum, we found three critical factors that challenged smooth progress from vision to curriculum reality.

Field-Test Feedback

The Core-Plus Mathematics authors worked very hard to create engaging investigations that would lead students to discovery of important mathematics. They received detailed and thoughtful feedback from the other strand authors and the overall coordinating author and produced numerous iterations of draft material, even before field tests. However, despite those best efforts, feedback from the field-test teachers often douses author enthusiasm with a cold shower of reports that individual problems or whole investigations just don't work. As discouraging as such reports sometimes are, they are essential to developing a curriculum that "works" in real schools.
This experience in the curriculum development process confirms two key points that are made about the design research process: "Development and research take place through continuous cycles of design, enactment, analysis, and redesign" (DBRC 2003 citing Cobb 2001 and Collins 1992), and "Research must account for how designs function in authentic settings. It must not only document success or failure but also focus on interactions that refine our understanding of the learning issues involved." (DBRC 2003)

In part, the often-discouraging reports from initial field tests reflect the reality that schools, classes, and teachers differ in very significant ways. So only quite robust curriculum designs and specific materials can be broadly useful without adaptation. However, they also highlight a particular challenge in writing materials that aim to support student-centered rather than teacher-directed instruction. We cannot say that we have found a way around this challenge of curriculum development, except to learn from feedback and to be creative in finding new approaches to the problematic topics.

Fidelity of Implementation

It is tempting to write off negative reports from field tests by guessing that the problematic material was simply not used as intended. In fact, there is a substantial challenge in fairly testing radically new ideas about classroom instruction and new mathematical goals for the curriculum. Field-test teachers often report puzzling over the level of mastery expected on topics in the new curriculum, because their experiential reference points have been knocked askew by the new content development. They also find it challenging to let students struggle a bit with open-end problem tasks. As a result, particularly in the first classroom testing of a new unit or course, it is unlikely that the material is taught as the author envisioned.

In response to this challenge of implementing radical change and giving the new ideas a fair chance to succeed, we have made a variety of efforts to be sure we are reacting to reports that do indeed reflect experience with the intended curriculum. We monitor the field-test sites by visiting them as often as possible (not often enough), and try to design field-test situations that give us a variety of contexts in which the material is being tested.

One of the most consistent reports from field tests is that there is more material in the curriculum than schools can reasonably teach. These reports can be tempered by other information (such as the sometimes astonishingly short class periods allowed for mathematics). But the reports of time pressure also give us alerts to places where we have been too ambitious and revision is required. Here again, the iterative cycle of writing, testing, and revision is critical to developing an effective curriculum package.

Design Principles Revisited

In 2002, the Core-Plus Mathematics Project received a five-year award from the National Science Foundation to prepare a second edition of the Core-Plus Mathematics curriculum. The revision was informed by research on the program's effectiveness, including a five-year longitudinal study (Schoen, Ziebarth, Hirsch, and BrckaLorenz, 2010), and by extensive feedback from teachers using the first edition texts. The revision also took into account changes in middle school mathematics programs, particularly in the area of algebra, the evolving nature of undergraduate mathematics, and advances in technology.

Content Sequence and Organization

The revision work resulted in some shifts in positioning and priorities of mathematical content in the four-year curriculum, most notably in the algebra and functions and geometry and trigonometry strands. In the case of the algebra and functions strand, changes resulted in:

- an accelerated introduction of symbol-based reasoning—including symbol manipulation and proof
- more explicit development of symbol sense—connecting algebraic forms to numeric, graphic, and context interpretation and implications
- earlier introduction of inverse functions and logarithms

In the case of the geometry and trigonometry strand, major content changes resulted in:

- a reorganized development of key geometric ideas—congruence, properties of circles, and trigonometric ratios and functions
- earlier and more careful development of geometric reasoning and proof
- increased use of coordinate representations and algebra to support geometric reasoning and problem solving

Instructional Design

The instructional materials themselves reflected the same design principles that guided development of the first edition, albeit with some refinements based on reports from first-edition users. Refinements included the development of focusing questions at the beginning of each lesson investigation to provide an advance organizer for the mathematics to be discovered or developed. For example, in a new Course 1 unit, *Quadratic Functions*, the introduction to an investigation in the second lesson includes:

As you work on the problems in this investigation, look for answers to these questions:

What strategies are useful in finding rules for quadratic functions?
In deciding when two quadratic expressions are equivalent?
In deciding when one type of quadratic expression is more useful than another?

A second refinement was the inclusion of Review exercises in each homework set designed to build proficiency with concepts and skills through distributed practice, to provide just-in-time review, or to address possible prerequisite gaps due to the more robust assumption about middle school preparation of students.

Role of Technology

Because of concerns for access and equity, the first edition curriculum materials were based on a modest technology assumption—students would have access to graphing calculators in class and outside of school. As work began on the second edition, the contextual and mathematical problems that the curriculum was being organized around and the learning expectations for students were such that it was desirable to augment graphing calculator technology with computer tools. To meet this challenge, and maintain the project's commitment to access and equity, the project systematically explored development of Java-based software that eventually evolved into *CPMP-Tools*—a suite of both general purpose and custom tools whose development continues to be informed by, and integrated with, the development of the curriculum materials.

- Tools were developed for each strand of the curriculum—algebra, geometry, statistics, and discrete mathematics.

    Algebra—The software for work on algebra problems includes a spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities.

    Geometry—The software for work on geometry problems includes an interactive drawing program for constructing, measuring, manipulating, and transforming geometric figures and a set of custom tools for studying geometric models of physical mechanisms, tessellations, and special shapes. The user has the options of working in a two- or three-dimensional environment, with or without coordinates.

    Statistics—The software for work on data analysis and probability problems provides tools for graphic display and analysis of data, simulation of probability experiments, and mathematical modeling of quantitative relationships.

Discrete Mathematics—The software for work on discrete mathematical modeling problems provides tools for constructing, manipulating, and analyzing vertex-edge graphs and networks.

- Developing student disposition and ability to make decisions about what technology tool to use and when was an important consideration. The design of *CPMP-Tools* keeps the possibilities up front.
- Tools and their functionality were organized by course to focus on the intended mathematics and to reduce the steepness of the learning curve. This has allowed the software capabilities to evolve with the mathematics and student understanding.
- Tools share similar menu screens and interface promoting learning transfer from one tool to another.
- Topic-specific tools are embedded within each strand as custom apps.
- Files are included that provide electronic copies of most data sets essential for work on problems in each *Core-Plus Mathematics* course.
- Tools are built using Java WebStart, which permits safe, easy, and reliable distribution of software and automatic updating across different types of computers.
- As Gnu-public license software, *CPMP-Tools* is available for use by teachers and students who are using curricula other than *Core-Plus Mathematics*. Equally important, it allows other curriculum developers to modify and enhance the software and build on toolkits (much like the custom apps in *CPMP-Tools*) tailored to their curricular goals, subject to the general guidelines of Gnu-public license software.

From the outset, our goal was to develop learner-centered software built and available through an open source license. Given the unknown future, an open source license would help to ensure that others could both maintain and build upon our work. Furthermore, by utilizing Java WebStart, we have situated the tools in a manner making them easily upgraded in the future. Such a delivery mechanism also supports the potential use of the software in libraries, at home, or any location where students have Internet access.

The software development has not been without its complications, issues, and unresolved questions. Initially, the extent of Java software available in classrooms, labs, and homes, the amount of in- or out-of-class use by either teachers or students, and our ability to adequately develop tools in conjunction with the primary efforts of writing materials were largely unknown. Through field testing, while these issues remain, our comfort with them has grown. Another issue that seems to have resolved itself is the reaction on the part of the publisher who has been positive and supportive even though the software itself cannot be sold. The publisher is committed to maintaining *CPMP-Tools*. Further information about the design and capabilities of this evolving curriculum-embedded software can be found in Hart et al. (2007).

Teacher Support

The close work with a smaller group of teachers during the development of the second edition along with information from the field over the previous 10 years and reviews of the first edition teacher support materials prompted enhancements to these materials. These enhancements were designed to better support teacher learning and effective implementation of the curriculum.

The enhancements focused on more clearly conveying the mathematical goals at the unit, lesson, and sometimes problem level. This includes helping teachers understand the level of proficiency expected at various points on the learning trajectories of key ideas. In addition, enhancements were developed to help teachers better understand how students might approach problems within the investigations. This was done through notes labeled: "Common Error," "Misconception," or more generally, "Instructional Note." In addition, newly developed "Differentiation" notes supported teachers in making curriculum and instructional modifications to meet the needs of all students.

In addition to continuing to support teacher learning of mathematics, we explored ways to make the teacher support materials more educative in terms of pedagogical practice. We drew on a design framework developed by Davis and Krajcik (2005). For example, to assist teachers in thinking more deeply about the lesson launches and the investigation summaries, discourse scenarios were developed to provide annotated examples of student and teacher discourse at these pivotal stages of a lesson. These scenarios were written in a manner that enabled teachers to "see" their own students in the discussions. Embedded in these scenarios were follow-up questions that teachers might choose to use during the discussion and occasional parenthetical remarks explicating teacher moves.

By carefully examining the student masters produced by field-test teachers, we were provided insights into teachers' decisions to scaffold certain investigations. This information along with teacher notes in annotated field-test units provided guidelines for developing some additional student activity masters for the second edition that assist students with organization of work without reducing the cognitive demand of the mathematical tasks or redirecting the goals of the investigation.

Development of the Second Edition

The curriculum development process for the second edition was similar to that used for the first edition of *Core-Plus Mathematics*. Each revised text was the product of a three-cycle of research and development, pilot testing and refinement, and field testing in an expanded set of schools and further refinement prior to publication. In addition to the careful reviews by teachers and the project advisory board, work in the revision project was advised and critiqued by a special panel of mathematical consultants. This panel reviewed and commented on units as they were being developed, tested, and refined. This modification of our original development process worked well and proved to be beneficial and rewarding for both developers and consultants (cf., Maurer and McCallum 2006).

Design of the CCSS Edition

Many of the recommendations of the Common Core State Standards for Mathematics in terms of content and mathematical practices had been incorporated in the previous editions of the program and refined over the last 20 years. We have maintained our international-like approach to curriculum organization and our problem-based, inquiry-oriented, and technology-rich approach to teaching and learning. Our attention to mathematical habits of mind as a unifying theme of the curriculum has been expanded to encompass and make explicit the CCSS mathematical practices (Table 1).

Table 1: Standards for Mathematical Practice

Mathematical Practices	
MP1:	Make sense of problems and persevere in solving them.
MP2:	Reason abstractly and quantitatively.
MP3:	Construct viable arguments and critique the reasoning of others.
MP4:	Model with mathematics.
MP5:	Use appropriate tools strategically.[1]
MP6:	Attend to precision.
MP7:	Look for and make use of structure.
MP8:	Look for and express regularity in repeated reasoning.

Modeling with Mathematics

Mathematical modeling has been a central and unifying theme of each unit of *Core-Plus Mathematics* since its initial design and development. Figure 1 illustrates the modeling process as developed in *Core-Plus Mathematics*. Depending on the unit, the model may be deterministic or stochastic and involve continuous or discrete quantities. Our approach to mathematical modeling and its connections to reasoning, sense making, and mathematical habits of mind have been refined with each new edition.

---

[1] To support development of MP5, the *Core-Plus Mathematics* program includes *CPMP-Tools*, a suite of freely available software including a computer algebra system (CAS), a spreadsheet, and dynamic geometry, statistics, and probability tools, together with custom apps specific to the curriculum.

In creating the CCSS edition of *Core-Plus Mathematics*, we have used mathematical modeling as an effective way of connecting the Mathematical Practices and the Content-related Standards across Conceptual Categories. As suggested in Figure 1, the process of mathematical modeling focuses throughout on making sense of problems in context (MP1) and using mathematics to model authentic problems in everyday life, society, and careers (MP4).

To model a real-world situation entails first representing the situation mathematically, drawing on mathematics in the content standards and flexibly using mathematical practices MP2, MP5, and MP7.

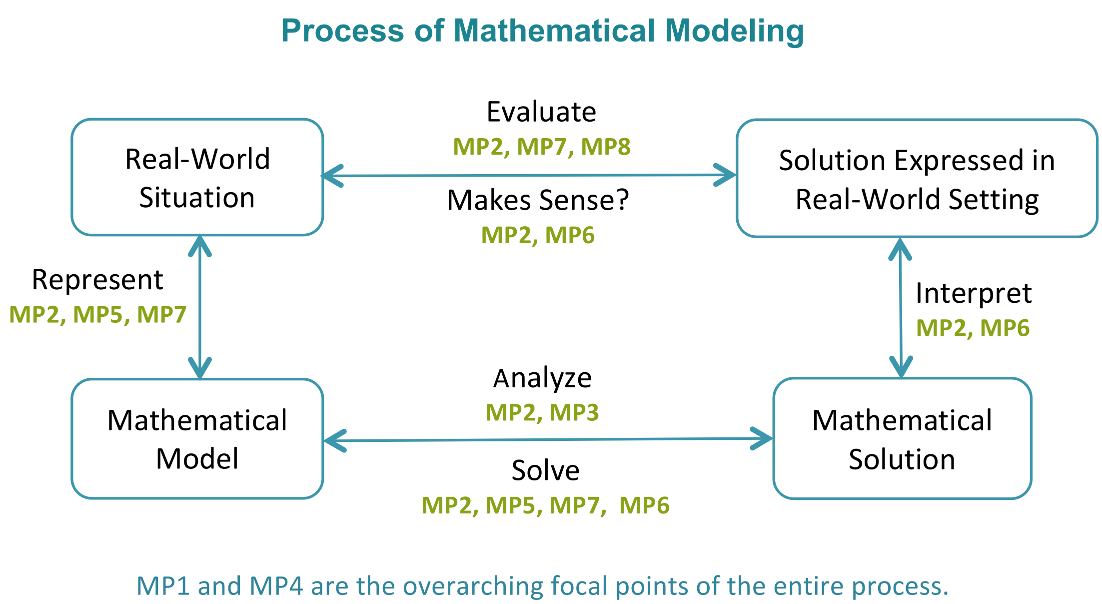

Figure 1: Connecting Mathematical Practices and the Content Standards

Once a mathematical model has been constructed, it is analyzed in terms of its faithfulness to the real-world situation and modifications may result from quantitative, spatial, or abstract reasoning about the “goodness of fit” (MP2), taking into account arguments provided by others (MP3).

Once a satisfactory model has been arrived at, a solution can be attempted. This solution process may involve further mathematical reasoning (MP2), strategic use of tools (MP5), making use of structure (MP7), and communicating precisely the results with attention to needed precision in calculations (MP6). Once a mathematical solution is determined, it must be interpreted back to the real-world situation again using practices MP2 and MP6.

After the solution is interpreted, answering the question “Does it make sense?” is essential and draws once again on MP2 and MP6. Looking back at, and evaluating the solution involves mathematical reasoning (MP2) and often leads to identification of mathematical structure is repeated calculations, algebraic manipulations, or reasoning patterns (MP7 and MP8).

In summary, the benefits of modeling with mathematics across a wide range of contexts lie not only in the solutions to the problems, but also in the new ideas and relationships that are discovered—the essence of the CCSS content standards.

CCSS Alignment

Aligning *Core-Plus Mathematics* with the CCSS has resulted in increased attention to some topics. For example, geometric transformations, their representations, properties, and applications have been elevated to a more central role in the curriculum and are now integrated throughout Courses 1–4. Similarly, we have refined our careful developmental progression of reasoning, justification, argumentation and proof across the four courses. Decreased attention has been given to discrete mathematics with a focus limited to discrete mathematical modeling. For further details on the CCSS alignment of the new edition, see *CCSS Guide to Core-Plus Mathematics* (Core-Plus Mathematics Project, 2015).

Course 4, originally designed as an advanced mathematical sciences course, has been redesigned as a *Preparation for Calculus* for college-bound STEM (science, technology, engineering, and mathematics) students. We have received a new grant from NSF to develop an alternative capstone course, *Transition to College Mathematics and Statistics*, for college- or career-bound students whose program of study or apprenticeship does not require calculus. This course shares many of the design features of the CCSS Edition of *Core-Plus Mathematics*.

Recognizing the challenge many teachers face in implementing the CCSS mathematical practices and content expectations in their classroom, expanded and enhanced Teacher's Guides have been prepared for each course to include:

- Unit openers providing guidance related to the CCSS and their implementation
- A CCSS Pathway and a CPMP Pathway through each unit
- A list of the CCSS indicators "focused on" and "connected to" for each lesson and for each investigation
- Additional margin notes that support instruction, differentiation, and the use of *CPMP-Tools* and other technologies
- Highlighted examples of problems and tasks that engage students in the CCSS mathematical practices

Finally, our CCSS-oriented curriculum work is also focusing on the design of highly interactive digital versions of the student and teacher editions for each course.

Summary

The conceptualization, development, and evaluation of the *Core-Plus Mathematics* curriculum materials has been a collaborative effort that blends the mathematical and pedagogical insights and practical experiences of mathematicians, mathematics curriculum and assessment developers, and classroom teachers. We believe that this collaboration provides a foundation of sound mathematical judgments, access to advice from the best of current thinking and research about learning and teaching, and the wisdom of practice that makes it possible for strong and appropriate new curriculum ideas to work in real school situations.

While there is contentious public discourse about school mathematics that suggests a disconnect of the key players in curriculum work, we've found that it is feasible and desirable to draw mathematicians, mathematics education researchers, and classroom teachers together in productive collaboration. Such collaboration requires time for extended planning dialogue about content and teaching issues and a development process that involves iterative cycles of design and field trials through which expertise of all parties can be applied to the task of materials production.

The use of curriculum materials in diverse classroom settings is a complex process that is shaped by influence of many different ideas, institutional traditions, and personal values. Thus it is not surprising that our own experience in developing the *Core-Plus Mathematics* program has been far from a smooth unfolding of designs that are logical consequences of mathematical, pedagogical, and design process principles. However, we believe that our operation under the umbrella of a generally shared set of such process principles has allowed us to produce a coherent and effective new approach to the mathematical education of high school students that ensures college and career readiness.

References

Brown, A. "Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings." *The Journal of the Learning Sciences* 2, no. 2 (1992): 141-78.

Burkhardt, Hugh, and Alan H. Schoenfeld. "Improving Educational Research: Toward a More Useful, More Influential, and Better-funded Enterprise." *Educational Researcher* 32, no. 9 (2003): 3-14.

Cobb, Paul. "Where Is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development." *Educational Researcher* 23, no. 7 (1995): 13-20.

Collins, A. "Toward a Design Science of Education." In *New Directions in Educational Technology,* edited by E. Scanlon and T. O'Shea. New York: Springer-Verlag, 1992.

Core-Plus Mathematics Project. *CCSS Guide to Core-Plus Mathematics*. Columbus, OH: McGraw-Hill Education, 2015.

Cuoco, Al, E. Paul Goldenberg, and June Mark. “Habits of Mind: An Organizing Principle for Mathematics Curricula.” *Journal of Mathematical Behavior* 15 (1996): 375-402.

Davidson, Neil, and D. L. Kroll. “An Overview of Research on Cooperative Learning Related to Mathematics.” *Journal for Research in Mathematics Education* 22, no. 5 (1991): 362-65.

Davis, Elizabeth A., and Joseph S. Krajcik. “Designing Educative Curriculum Materials to Promote Teacher Learning.” *Educational Researcher* 34, no. 3 (2005): 3–14.

The Design-Based Research Collective. “Design-based Research: An Emerging Paradigm for Educational Inquiry.” *Educational Researcher* 32, no. 1 (2003): 5-8.

Fey, James, and Christian Hirsch. “The Case of *Core-Plus Mathematics*.” In *Perspectives on the Design and Development of School Mathematics Curricula*, edited by C. Hirsch. Reston, VA: National Council of Teachers of Mathematics, 2007.

Freudenthal, H. *Didactical Phenomenology of Mathematical Structures*. Dordrecht, The Netherlands: D. Reidel, 1983.

Gravemeijer, K. “Educational Development and Developmental Research in Mathematics Education.” *Journal for Research in Mathematics Education* 35, no. 5 (1994): 443-71.

Hart, Eric W., Christian R. Hirsch, and Sabrina A. Keller. “Amplifying Student Learning in Mathematics Using Curriculum-Embedded Java-based Software.” In *The Learning of Mathematics*, 69th yearbook of the National Council of Teachers of Mathematics, edited by W. G. Martin and M. E. Strutchens. Reston, Va.: NCTM, 2007.

Hiebert, James, Thomas P. Carpenter, Elizabeth Fennema, Karen Fuson, P. Human, H. Murray, A. Olivier, and Diana Wearne. “Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics.” *Educational Research* 25, no. 4 (1996): 12-21.

Hirsch, Christian (Ed.). *Perspectives on the Design and Development of School Mathematics Curricula*. Reston, VA: National Council of Teachers of Mathematics, 2007.

Maurer, Stephen B. and William McCallum. “Advising a Precollege Curriculum Project.” *Notices of the AMS* 53, no. 9 (2006): 1018–20.

National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, D.C.: National Academy Press, 1989.

Oakes, Jeannie. “Opportunities, Achievement, and Choice: Women and Minority Students in Science and Mathematics.” In *Review of Research in Education, 16,* edited by C. B. Cozden. Washington, D.C.: American Education Research Association, 1990.

Schoen, Harold L., and Christian R. Hirsch. “The Core-Plus Mathematics Project: Perspectives and Student Achievement.” In *Standards-based School Mathematics Curricula: What Are They? What Do Students Learn?* edited by Sharon L. Senk and Denisse R. Thompson, pp. 311-43. Mahwah, N.J.: Lawrence Erlbaum, 2003.

Schoen, Harold L., Steven W. Ziebarth, Christian R. Hirsch, and Allison BrckaLorenz. *A Five-year Study of the First Edition of the Core-Plus Mathematics Curriculum*. Charlotte, NC: Information Age Publishing, 2010.

Steen, Lynn Arthur (ed.). *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, D.C.: National Academy Press, 1990.